TECHNICAL REPORT M-71-5

PERFORMANCE OF SOILS UNDER TRACK LOADS

Report 2

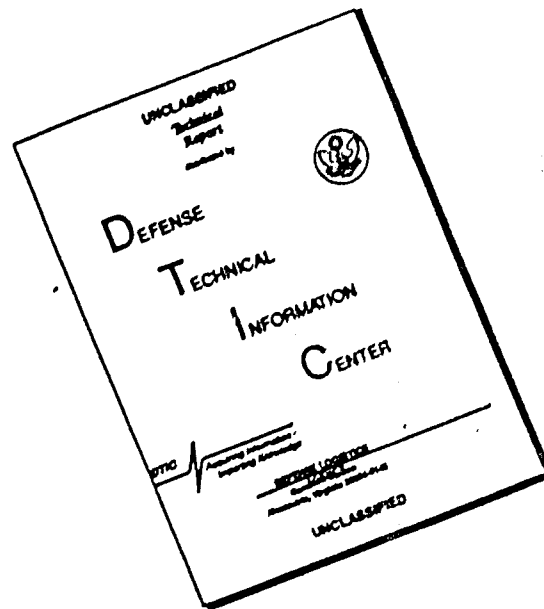## PREDICTION OF TRACK PULL PERFORMANCE IN A DESERT SAND

by

G. W. Turnage

November 1971

Sponsored by U. S. Army Materiel Command

Conducted by U. S. Army Engineer Waterways Experiment Station, Vicksburg, Mississippi

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13. ABSTRACT

A first-generation, quantitative description of straight-line track pull performance in a desert sand was sought by laboratory tests of a model track. Of 16 variables selected to provide a comprehensive description of the track-sand system, analysis of three Plackett-Burman (statistical) test designs showed four to merit initial study--soil strength G, track width b, track contact length ℓ, and load W. Principles of similitude were used to develop three dimensionless, independent terms--P_{20}/W, b/ℓ, and $G\ell^3/W$--to express the functional relation among these variables and P_{20} (pull at 20 percent slip, i.e. near-maximum pull). Data analysis demonstrated that the last two terms can be combined to $G(b\ell)^{3/2}/W$. A simple linear regression determined the least-squares fit of P_{20}/W to log $[G(b\ell)^{3/2}/W]$ as $P_{20}/W = 0.204482 + 0.161470$ log $[G(b\ell)^{3/2}/W]$. Values of a_0 and a_1 in $P_{20}/W = a_0 + a_1$ log $[G(b\ell)^{3/2}/W]$ were relatively stable in regression of data subgroups separated by the levels at which G, b, ℓ, and W were tested. Comparisons of linear regressions of form $P_{20}/W = a_0 + a_1$ log $\left(Gb^{x_1}\ell^{x_2}/W\right)$ (with $x_1 + x_2 = 3.0$ and values of x_1 and x_2 varying from 0 to 3) also indicated the optimum prediction term to be $G(b\ell)^{3/2}/W$. Finally, the equation from a multiple linear regression of P_{20}/W on log G, log b, log ℓ, log W did not describe the test data significantly better than did the simple linear regression. From equation $P_{20}/W = a_0 + a_1$ log $[G(b\ell)^{3/2}/W]$, expressions were obtained for P_{20}, W, and P_{20}/W in terms of a_0, a_1, k, and e, where $k = G(b\ell)^{3/2}$, for two performance levels of particular interest, the maximum-pull and zero-pull conditions.

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Deserts						
Sands						
Tracked vehicles						
Trafficability						
Vehicle performance						

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FOREWORD

This report was prepared by Mr. Gerald W. Turnage of the Research Projects Group, Mobility Research Branch, Mobility and Environmental Division, U. S. Army Engineer Waterways Experiment Station. The report is essentially a thesis submitted by Mr. Turnage in partial fulfillment of the requirements for the degree of Master of Science in Statistics to the Faculty of Florida State University, and is a study concerned with the statistical evaluation of results of tests with a single model track. The study described herein was conducted under DA Project 1T062103A046, "Trafficability and Mobility Research," Task 03, "Mobility Fundamentals and Model Studies," under the sponsorship and guidance of the Research, Development and Engineering Directorate, U. S. Army Materiel Command. The study was accomplished under the general direction of Messrs. W. G. Shockley and S. J. Knight, Chief and Assistant Chief, respectively, of the Mobility and Environmental Division.

COL Levi A. Brown, CE, and COL Ernest D. Peixotto, CE, were Directors of the Waterways Experiment Station during the period of preparation and publication of this report. Mr. F. R. Brown was Technical Director.

THE FLORIDA STATE UNIVERSITY

COLLEGE OF ARTS AND SCIENCES

PREDICTION OF TRACK PULL PERFORMANCE
IN A DESERT SAND

By

GERALD W. TURNAGE

A Thesis
Submitted to the Department of Statistics
in partial fulfillment of the requirements
for the degree of Master of Science

Approved:

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Professor Directing Thesis

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August, 1971

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The author wishes to express his appreciation to Dr. D. A. Meeter for his guidance and encouragement throughout this research. Appreciation is also extended to the author's supervisors of the U. S. Army Engineer Waterways Experiment Station, particularly Messrs. S. J. Knight and W. G. Shockley, for their staunch support of ongoing education for their subordinates. Special acknowledgment is offered Miss M. E. Smith, mathematician; Mr. R. B. Ahlvin, computer operator; and Mrs. Jane N. Brown, secretary, all of whose assistance in the preparation of this thesis was invaluable.

Last, but not least, the author expresses his sincere appreciation to his wife, ███████████████████████████ for the patience and understanding they exhibited during this study and to whom this work is dedicated.

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CHAPTER I

STATEMENT OF THE PROBLEM

No comprehensive, test-proven system exists today for predicting in-soil tracked vehicle performance quantitatively and with good accuracy. This is not too surprising when one considers the complexity of the interaction of rotating, slipping, geometrically complex tracks of variable tension and weight distribution with soils having a wide variety of physical properties. The number of terms required to provide a complete description of soil-track interactions is quite large, and the task of developing a theoretically sound charaterization of all major soil-track interactions is staggering.

The need for a quantitative description of the soil-track system certainly exists. At present, the design of tracked vehicles is based on well-defined knowledge of the operating characteristics of the various components of the vehicle, but on only vague knowledge of the requirements imposed on the vehicle by the environment in which it must operate, i.e. by off-road conditions. Rational procedure requires that a strong effort be made to provide the tracked-vehicle designer with a useful description of the soil-track system. A first-generation description can best be developed through analysis of data obtained in a laboratory environment, where close control and systematic programming of values of the soil and track variables are possible. A

1

photograph of the model track, dynamometer carriage, and soil test pit
used at the U. S. Army Engineer Waterways Experiment Station (WES) in
the study of soil-track interactions is shown in Figure 1.1. Because
the number of terms needed to provide a full description of all types
of track behavior in all types of soils is much too large to deal with
in a single investigation, an overall description must be obtained in
parts. As a first step, this thesis will develop a technique for pre-
dicting the pull that a track develops while operating in a straight

FRONT VIEW OF MODEL TRACK MOUNTED IN LARGE-SCALE DYNAMOMETER TEST CARRIAGE
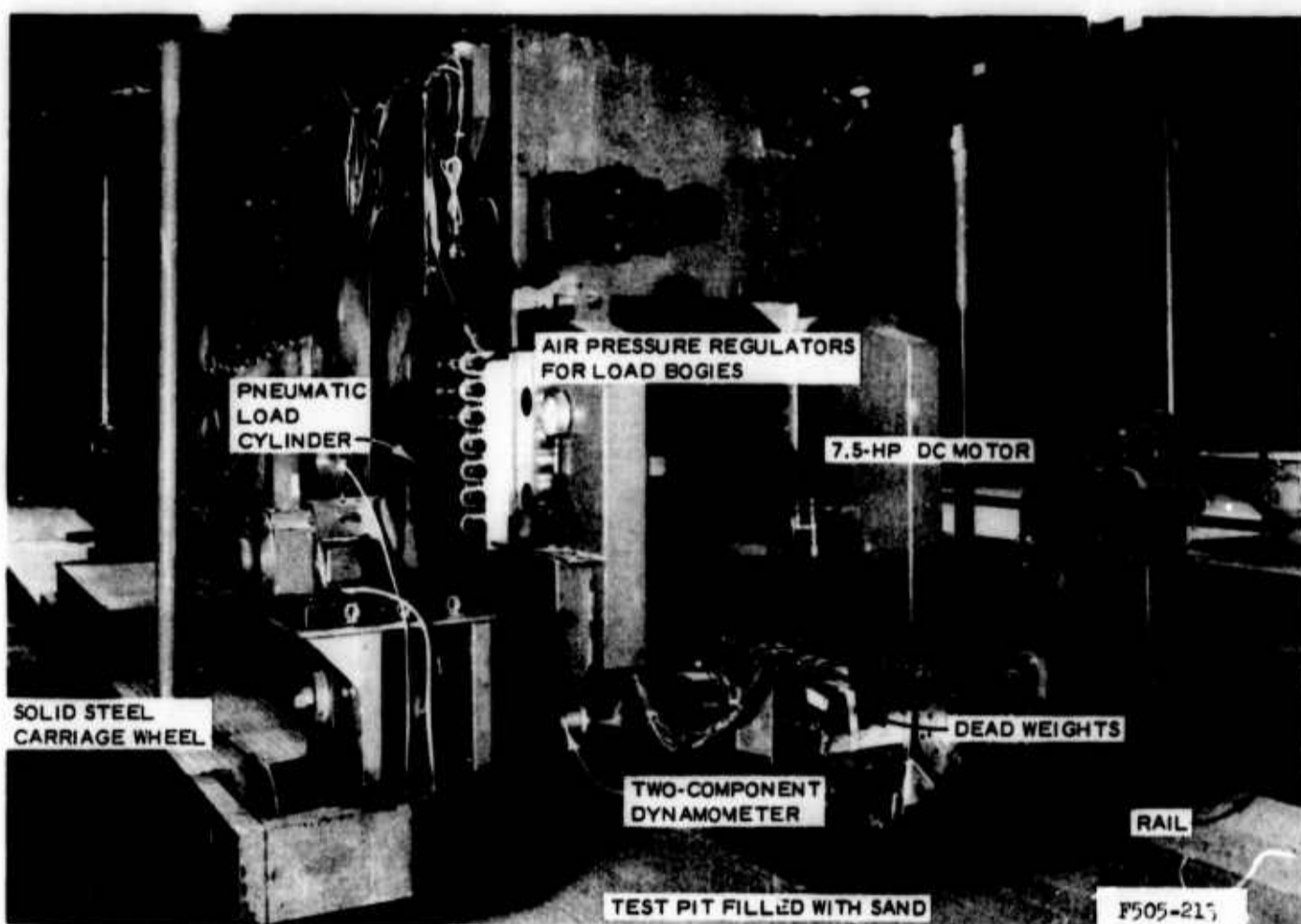
FIGURE 1.1

RELATION BETWEEN PULL AND SLIP FOR TRACKS OPERATING STRAIGHT-LINE IN LEVEL SECTIONS OF AIR-DRY DESERT SAND
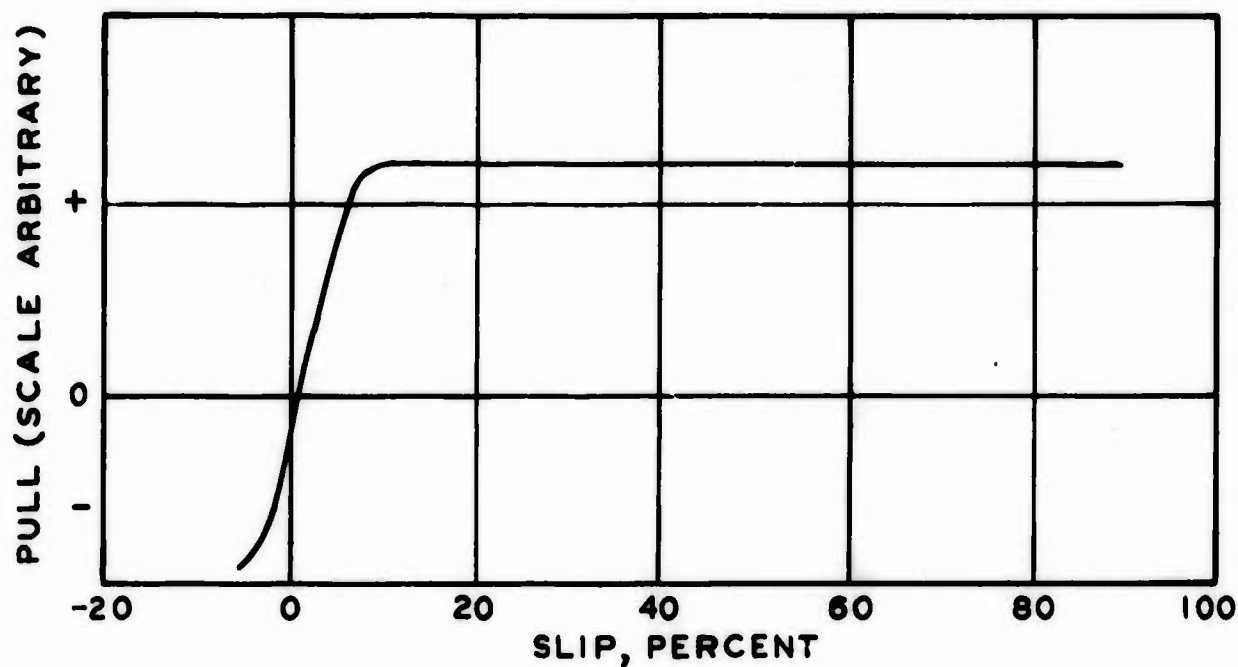
FIGURE 1.2

line (i.e. no maneuvering) at near-optimum slip (20 percent slip--see Figure 1.2) in a level test bed of air-dry desert sand (Yuma sand). (Slip is defined on page 8.) Hereafter, pull at 20 percent slip will be designated P_{20} . The study will include:

(i)   The determination of the soil and track variables that influence track pull most in a desert sand.

(ii). The development of dimensionless terms composed of functions of the most important independent soil and track variables, and manipulation of these terms to produce a single dimensionless prediction term.

(iii) Application of statistical techniques to determine the functional relation between the prediction term and dimensionless pull (i.e. pull/load), and to examine in detail the adequacy of this relation.

CHAPTER II

DETERMINATION OF MOST IMPORTANT INDEPENDENT VARIABLES

A. Independent variables considered

Seventeen independent (controlled) variables were selected to provide a reasonably comprehensive description of the soil-track system for track pull performance at 20 percent slip in air-dry Yuma sand. These variables and the ranges of values that can be obtained for each with the WES test equipment are listed in Table 2.1. Only 16 of

TABLE 2.1

INDEPENDENT SOIL-TRACK VARIABLES CONSIDERED AND

RANGES OF POSSIBLE VALUES FOR EACH

Variable	Range of Values
Load, W	Near zero to 27 kN*
Soil strength, G	0.5 to 7.5 MN/m^3** (approximately)
Track-ground contact width, b	15.2, 30.5, and 61.0 cm
Nominal track-ground contact length, ℓ (on a hard surface)	61.0 and 121.9 cm
Angle of approach, α	5.5 to 18.5 deg with forward-end bogie fully retracted; 21.5 to 33.0 deg with it fully extended
Angle of departure, β	Same ranges as for α (based on positions of rear-end bogie)

TABLE 2.1 (Concluded)

Variable	Range of Values
Road-wheel diameter	17.8 cm only with present system
Road-wheel spacing (uniform with at least one inner road wheel)	For ℓ = 61.0 cm: 20.3 cm only For ℓ = 121.9 cm: 20.3, 40.6, and 61.0 cm
Pressure (pneumatic) in road bogies	0 to 690 kN/m^2 ($\approx$620 kN/m^2 = maximum usable)
Distribution of pressure in bogies	Wide variety possible
Location of drive sprocket	Front or rear (can be described in terms of geometric location relative to some fixed point on the track)
Track-shoe height	1.3, 2.5, and 5.1 cm
Track-shoe thickness	0.32 and 0.64 cm
Track-shoe spacing	3.0 cm (all shoes in); 14.2 cm (every other shoe removed)
Index of track-belt tension	Gage range = 0 to 20, 700 kN/m^2; values of 1380 to 6900 kN/m^2 have been tested (see (v) on page 6)
Translational velocity	0 to 0.6 m/sec
Track-frame trim angle, θ	0 to 20 deg has been tested

* Kilonewtons.
** Meganewtons per cubic meter.

these variables were considered in this analysis; road-wheel diameter was eliminated because it could be tested at only one value, 17.8 cm. Each of these variables and the means used to control its value are described by Turnage [1971]. Track geometry terms are illustrated in

Figure 2.1, where schematics of the overall model track and of a section of WES band-type track are presented. Variables not shown in Figure 2.1 are described below:

(i) <u>Index of soil strength, G</u>. For essentially cohesionless soils, penetration resistance gradient G characterizes soil strength as the average slope of the near-linear curve of cone penetration resistance versus depth of penetration. Cone penetration resistance is the force per unit base area required to move a 30-deg right circular cone of 3.23-cm^2 base area through the soil normal to the soil surface at a rate of 3.05 cm/sec. G is measured in that depth range for which changes in soil strength noticeably affect the performance of a track (taken as 0 to 15 cm in this study).

(ii) <u>Load, W</u>. Overall test load W includes the weight of the model track plus load applied at the load axle.

(iii) <u>Pressure (pneumatic) in road bogies</u>. The road bogies, taken together, support the overall test load. Each road bogie includes a pneumatic cylinder whose pressure is individually regulated and monitored.

(iv) <u>Distribution of pressure in road bogies</u>. Because pressure in the road bogies is individually regulated, an unlimited variety of pressure distributions in the road bogies is possible.

(v) <u>Index of track-belt tension</u>. This index is obtained from measurement of the pressure that a regulated hydraulic jack exerts in an outward direction on the track's idler sprocket.

(vi) <u>Translational velocity</u>. Actual rate of track horizontal advance. The travel ratio is the ratio of translational velocity to

SCHEMATICS OF THE WES MODEL TRACK

a. OVERALL SIDE VIEW

TOP VIEW
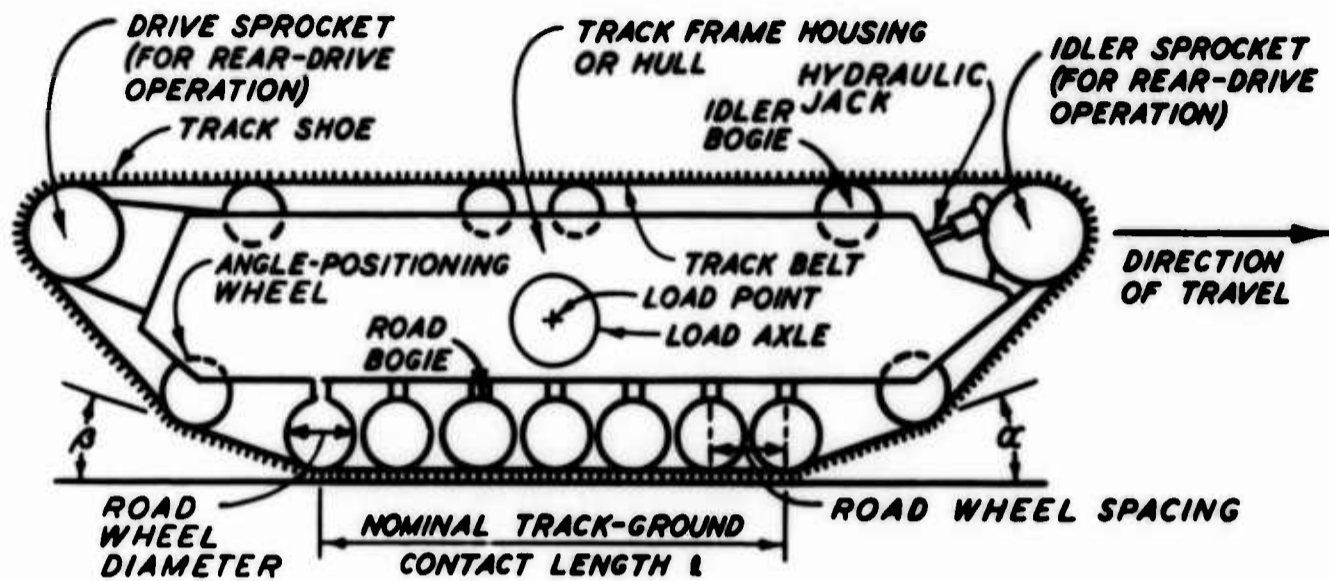
SIDE VIEW

END VIEW
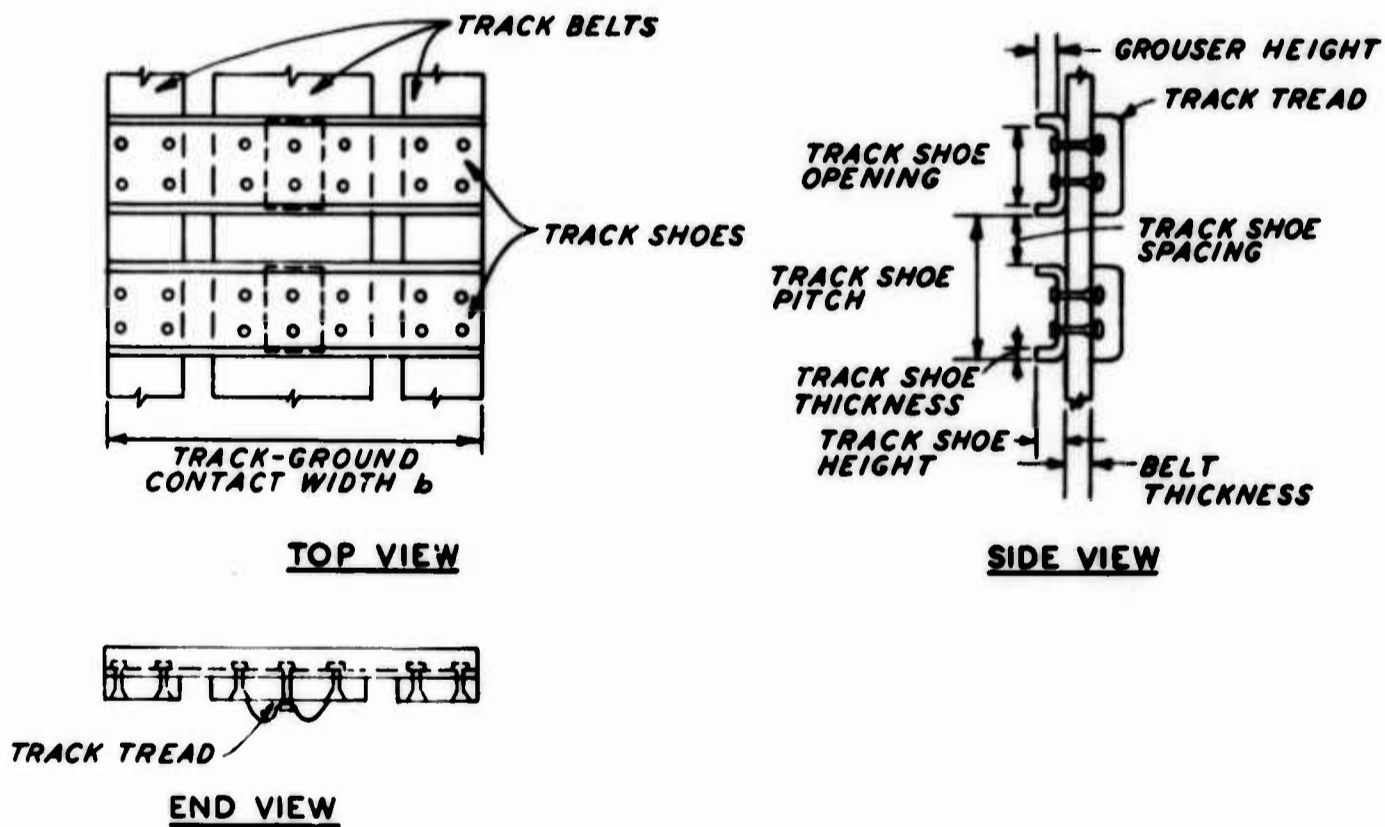
b. SECTION OF WES MODEL BAND-TYPE TRACK

FIGURE 2.1

the theoretical rate of horizontal advance (taken as $r\omega \cos \epsilon$, where r is track drive radius, ω is the angular velocity of the drive sprocket, and ϵ is the angle between the bottom of the track and a horizontal plane). Slip is defined as unity minus the travel ratio.

(vii) Track-frame trim angle, θ. The angle between the bottom of the track frame and the original soil surface.

B. Screening important variables by Plackett-Burman analysis

Analysis of track pull performance is greatly simplified by first determining which of the 16 independent soil-track variables have the most influence on track pull, and then restricting the description of the soil-track system to that provided by the most important independent variables. To identify the most important variables, a screening design of the type developed by R. L. Plackett and J. P. Burman [1946] was used. Interest centered primarily in determining those variables whose main effects on track pull are greatest, and in accomplishing this with a minimum of testing. These requirements suggested use of a design in which each variable is tested at only two values. The Plackett-Burman designs drastically reduce the number of tests required by a 2^p factorial design at the expense of a high degree of fractionation. Main effects are not confounded with each other, but are confounded with two-factor and higher order interactions. Independence of estimated effects is maintained in the design, and an estimate of the variance of the test variables is provided by a pooled estimate of variance of dummy variables.

Several Plackett-Burman designs are illustrated in Figure 2.2. The size of the matrix is indicated by the letter n, where n

SOME PLACKETT-BURMAN MATRICES

TEST* NO., n	A	B	C	D	(E)**	(F)	(G)
1	+	+	+	-	+	-	-
2	+	+	-	+	-	-	+
3	+	-	+	-	-	+	+
4	-	+	-	-	+	+	+
5	+	-	-	+	+	+	-
6	-	-	+	+	+	-	+
7	-	+	+	+	-	+	-
8	-	-	-	-	-	-	-

a. R = 4 REAL VARIABLES; n = R + 4 = 8 TESTS

TEST NO., n	A	B	C	D	E	F	G	H	(I)	(J)	(K)
1	+	+	-	+	+	+	-	-	-	+	-
2	+	-	+	+	+	-	-	-	+	-	+
3	-	+	+	+	-	-	-	+	-	+	+
4	+	+	+	-	-	-	+	-	+	+	-
5	+	+	-	-	-	+	-	+	+	-	+
6	+	-	-	-	+	-	+	+	-	+	+
7	-	-	-	+	-	+	+	-	+	+	+
8	-	-	+	-	+	+	-	+	+	+	-
9	-	+	-	+	+	-	+	+	+	-	-
10	+	-	+	+	-	+	+	+	-	-	-
11	-	+	+	-	+	+	+	-	-	-	+
12	-	-	-	-	-	-	-	-	-	-	-

b. R = 8 REAL VARIABLES; n = R + 4 = 12 TESTS

```
n = 12   + + - + + + - - - + -
n = 16   + + + + - + - + + - - + - - -
n = 20   + + - - + + + + - + - + - - - - + + -
n = 24   + + + + + - + - + + - - + + - - + - + - - - -
n = 28   + - + + + + - - - - + - - - + - - + + + - + - + + - +
```

c. FIRST ROW OF PLACKETT-BURMAN MATRIX FOR SEVERAL VALUES OF n

- = LOW LEVEL
+ = HIGH LEVEL

* TESTS ARE RUN IN RANDOM ORDER

** PARENTHESES AROUND A LETTER INDICATE A DUMMY VARIABLE. NO RE-
STRICTION IS PLACED ON WHICH COLUMNS MAY BE CHOSEN TO REPRESENT
DUMMY VARIABLES.

FIGURE 2.2

identifies the number of rows, and n - 1 is the number of columns.

Plackett-Burman matrices have been developed for variables tested at two

levels for n = 8, 12, 16, ..., 84, 88 (92 missing), 96, 100. Each col-

umn heading without parentheses designates an actual variable of the

test system. A column heading with parentheses names a dummy (ficti-

tious) variable; + and - signs for dummy variables are ignored when

the actual testing is done. Entries in each row represent the values

of the variables associated with one test. Each real variable is

assigned one practical high-level value (designated by a +) and one

practical low-level value (indicated by a -) at which it can be main-

tained nearly constant during testing. The sign is consistent along

any diagonal from right to left beginning at any location along either

the top row or right-most column, except that the last row has all

minus signs. Plackett-Burman matrices for all values of n have these

characteristics, so that any particular Plackett-Burman matrix is speci-

fied when only its first row is known. The first rows of matrices for

several values of n are shown at the bottom of Figure 2.2.

Ideally, a potentially complex system should be examined at the

inception of its study through use of a Plackett-Burman matrix of a

size just sufficient to treat all possibly important system variables.

If R equals the number of system (i.e. not dummy) variables, this re-

quires at least $n = R + 4$ tests. This results because analysis of

test data is based on a t-test of the difference between mean values of

the test response when the variable of interest is tested at its low-

level value and when it is tested at its high-level value. For a given

level of significance, the percentage points of t decrease very

quickly as the number of degrees of freedom increase from one to three, and they continue to decrease much more slowly thereafter. Thus, the power of the t-test (i.e. the probability of rejecting the null hypothesis) is greatly improved if at least three degrees of freedom (i.e. three dummy variables) are present in the test program, or if no more than $n - 4$ of the variables considered are system parameters.

Unfortunately, several factors associated with the physical testing of the model track did not allow testing all 16 system variables with one Plackett-Burman design. During the early stages of testing, WES had available track shoes of only one height, 2.54 cm. Road-wheel spacing could be doubled for the long track configuration ($\ell = 121.9$ cm), since it included seven road wheels and an even number of spacings; but spacing could not be doubled for the short configuration ($\ell = 61.0$ cm) because it had four road wheels and an odd number of openings. Also, corresponding patterns of road-bogie pressure distributions (other than uniform) cannot be obtained for tracks of different lengths. Finally, the chance of obtaining a large t-test value for a particular variable increases directly with the size of the difference between high- and low-level values at which the variable is tested (if constant variance of the system variables is assumed). Obvious interactions among several of the 16 system variables (load and trim angle, for instance) restrict the range of + and - values possible for the interacting variables if they are tested together. This restrictive influence is fairly great for a design that requires a different combination of 16 + and - signs in each of 20 different tests.

Ultimately, the effects of the 16 real soil-track variables of

Table 2.1 (road-wheel diameter was omitted) were studied in three separate Plackett-Burman designs. Unless it was specified as one of the variables whose effect on P_{20} was being studied, each variable had its value maintained constant at the level indicated in Table 2.2. The

TABLE 2.2

VALUES OF THE 16 SAND-TRACK SYSTEM VARIABLES WHEN THEY WERE

HELD CONSTANT IN A PLACKETT-BURMAN DESIGN

Variable Name	Constant Value		
	Design No. 1	Design No. 2	Design No. 3
Load, W	--	4450 N*	10,000 N
Soil strength, G	--	1.5 MN/m^3	3.3 MN/m^3
Track-ground contact width, b	--	15.2 cm	15.2 cm
Track-ground contact length, ℓ (on a hard surface)	--	121.9 cm	121.9 cm
Angle of approach, α	22 deg	--	22 deg
Angle of departure, β	22 deg	--	22 deg
Road-wheel spacing (uniform with at least one inner road wheel)	20.3 cm	20.3 cm	--
Pressure (pneumatic) in road bogies	276 kN/m^2	--	621 kN/m^2
Distribution of pressure in road bogies	Uniform	--	Uniform
Location of drive sprocket	Rear	Rear	--
Track-shoe height	2.54 cm	2.54 cm	--
Track-shoe thickness	0.32 cm	--	0.32 cm
Track-shoe spacing	3.0 cm (all in)	--	3.0 cm (all in)
Index of track-belt tension	4140 kN/m^2	--	6890 kN/m^2
Translational velocity	0.6 m/sec	--	0.6 m/sec
Track-frame trim angle, θ	Unrestrained	Unrestrained	--

* Newtons.

16 system variables were assigned the low- and high-level values listed in Table 2.3 when they were tested in three Plackett-Burman designs.

TABLE 2.3

LOW-LEVEL (-) AND HIGH-LEVEL (+) VALUES ASSIGNED TO THE 16 SAND-

TRACK SYSTEM VARIABLES IN THREE PLACKETT-BURMAN DESIGNS

Real Variable Designation	System Variable Name	Low-Level Value (-)	High-Level Value (+)
	Plackett-Burman Design No. 1		
A	Soil strength, G	1.5 MN/m^3	3.0 MN/m^3
B	Load, W	3110 N	6230 N
C	Track width, b	15.2 cm	61.0 cm
D	Track length, ℓ	61.0 cm	121.9 cm
	Plackett-Burman Design No. 2		
A	Angle of approach, α	22 deg	30 deg
B	Angle of departure, β	22 deg	30 deg
C	Translational velocity	0.3 m/sec	0.6 m/sec
D	Index of track-belt tension	1380 kN/m^2	4140 kN/m^2
E	Track-shoe spacing	3.0 cm	14.2 cm
F	Track-shoe thickness	0.32 cm	0.64 cm
G	Minimum pressure in road bogies	103 kN/m^2	241 kN/m^2
H	Distribution of pressure in road bogies	Same for all bogies	Uniform increase, rear to front
	Plackett-Burman Design No. 3		
A	Track-shoe height	1.3 cm	5.1 cm
B	Road-wheel spacing	20.3 cm	40.6 cm
C	Drive-sprocket location	Front	Rear
D	Track-frame trim angle	0 deg	20 deg

Both the first and third Plackett-Burman designs were based on
the matrix in part a of Figure 2.2; the second design was based on the
matrix in part b of Figure 2.2. Values obtained for track pull at

20 percent slip (P_{20}) in the three Plackett-Burman designs are presented in Table 2.4.

TABLE 2.4

VALUES OF TRACK PULL AT 20 PERCENT SLIP OBTAINED

IN THREE PLACKETT-BURMAN DESIGNS (NEWTONS)

Plackett-Burman Design No.	Test No.											
	1	2	3	4	5	6	7	8	9	10	11	12
1	4777	2820	2415	3083	1490	1802	3430	1112	--	--	--	--
2	2318	2349	2509	1824	2064	1744	1966	2251	1957	1984	2028	2340
3	3483	283	3796	3123	2187	1693	826	3549	--	--	--	--

The effect on the test response of each variable (both system and dummy) is defined as

$$\text{Effect on } y = \sum_{i=1}^{n} \frac{y_i s_i}{n/2} \qquad [2.B.1]$$

where

y_i = the measured value of the test response (P_{20}) in the ith test

s_i = +1 if the variable of interest is + in the ith test, -1 if the variable of interest is - in the ith test

n = total number of tests in the design of interest

The variance of the effects is estimated from Stowe and Mayer [1966] by

$$V(\text{effect on } y) = \frac{\sum_{i=1}^{m} (\text{Effect of dummy parameter})_i^2}{m} \qquad [2.B.2]$$

where m = the number of dummy variables . A value of the $\underline{t}$-statistic to test for the significance of a particular effect is given by $\underline{t} = \dfrac{\text{effect}}{\text{S.E.}}$, where S.E. is the estimate of the standard error of the effect provided by the square root of the variance defined above. Estimates of effects, standard errors, and $\underline{t}$-values (each based on three degrees of freedom) are presented i Table 2.5. Entries for the dummy variables are included in Table 2.5 primarily because their effects

TABLE 2.5

EFFECTS, STANDARD ERRORS, AND $\underline{t}$-VALUES OBTAINED

IN THREE PLACKETT-BURMAN DESIGNS

Plackett-Burman Design Number								
1			2			3		
Variable	Effect on P_{20} N	$\underline{t}$-Value	Variable	Effect on P_{20} N	$\underline{t}$-Value	Variable	Effect on P_{20} N	$\underline{t}$-Value
A	519	2.334	A	-128	-2.522	A	139	0.384
B	1823	8.197	B	11	0.217	B	-877	-2.421
C	980	4.406	C	93	1.833	C	164	0.453
D	461	2.073	D	139	2.739	D	-2241	-6.185
(E)	344	1.547	E	-4	-0.079	(E)	508	1.402
(F)	-23	-0.103	F	-22	-0.433	(F)	231	0.638
(G)	-172	-0.773	G	-385	-7.586	(G)	-287	-0.792
			H	-56	-1.103			
			(I)	-86	-1.695			
			(J)	-18	-0.355			
			(K)	-2	-0.039			
Estimate of standard error = 222.4 N			Estimate of standard error = 50.75 N			Estimate of standard error = 362.3 N		

are useful in the following analysis.

Comparison of results within each design can also be made

visually by using a half-normal plot of the type proposed by Daniel
[1959]. In Figure 2.3, the absolute value of effect is plotted against
the appropriate percentile value of the half-normal distribution for all
system and dummy variables of the three Plackett-Burman designs. Ac-
cording to theory, if the effects are due solely to a normally distrib-
uted random error, they should tend to fall along a straight line when
the cumulative empirical distribution is plotted in this manner. De-
viation from a straight line suggests significance. In design 1, vari-
ables A, B, and C are singled out for attention; in design 2, variable G;
and in design 3, variables B and D. The use of half-normal plot analy-
sis to interpret results from designs that estimate variance of the
system variables by means of only three dummy variables is quite

HALF-NORMAL PLOT OF EFFECTS FROM THREE PLACKETT-BURMAN DESIGNS
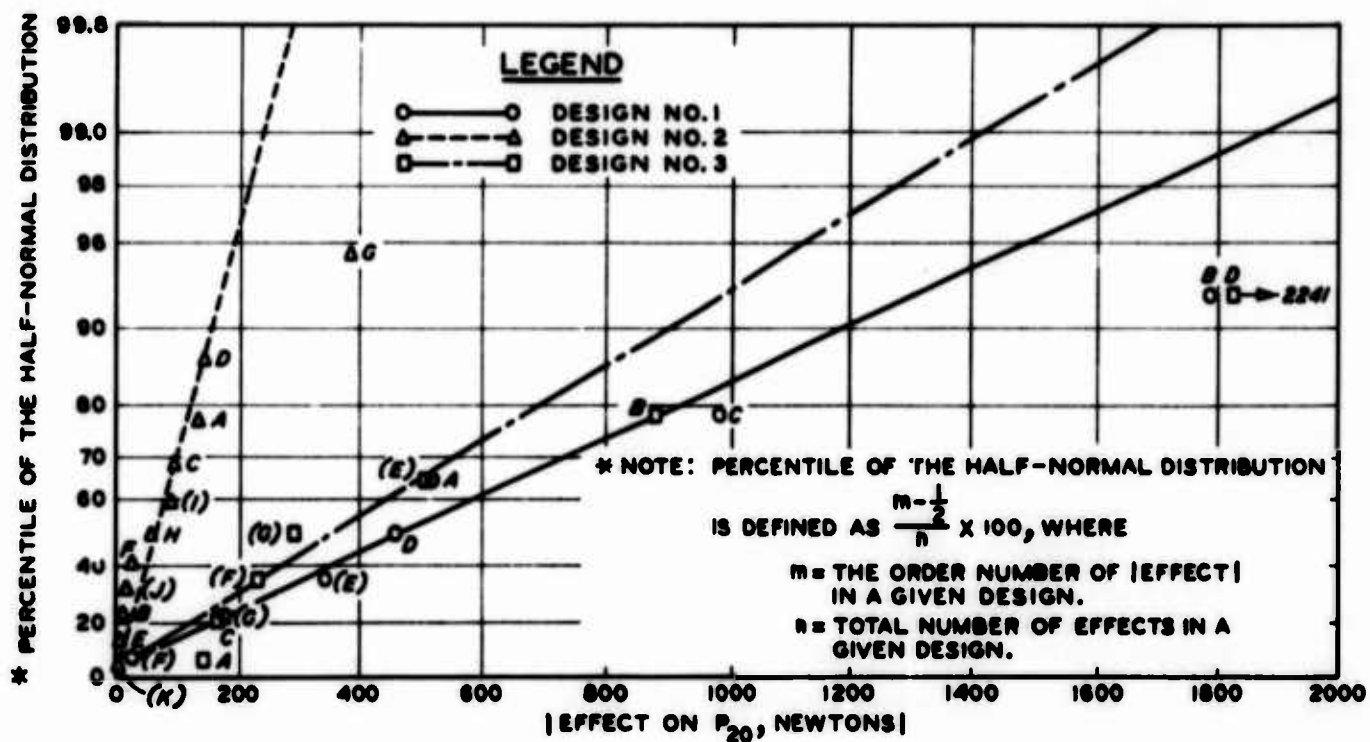
FIGURE 2.3

perilous; additional analysis of the test results is called for.

Comparison of the significance of effects obtained in different Plackett-Burman designs on the basis of t-values, is straightforward only if the variance of the system variables takes a common value in each design. Comparison of the ratio $\dfrac{(S.E._{max})^2}{(S.E._{min})^2} = 51.0$ for designs 1, 2, and 3 of Table 2.5 against tabled $F_{max} = 27.8$ for $\alpha = 0.05$ and three degrees of freedom (from Pearson and Hartley [1958]) indicates at the 5 percent significance level that differences do exist among variances in at least two of the designs. Each estimate of standard error is valid only insofar as the assumption is satisfied that certain higher order interactions are zero. Thus, a larger value of S.E. for a given design may indicate that interactions of system variables of that design exert more influences on the test response than do inter-actions of variables of a design with smaller S.E. , all other factors being equal. It is not necessarily reasonable, then, to scale increasing importance of system variables from different designs according to increasing values of the t-statistic.

If it is hypothesized that (a) variance of the effects, unaf-fected by interactions of the system variables, is relatively constant for all three designs, and (b) the smallest estimate of S.E. from the three designs reflects the standard error not contaminated by interac-tions, then t-values for the system variables are produced as shown in Table 2.6:

TABLE 2.6

t-VALUES FOR THREE PLACKETT-BURMAN DESIGNS

WITH STANDARD ERROR ESTIMATED AT 50.75 N

Plackett-Burman Design Number					
1		2		3	
Variable	t-Value	Variable	t-Value	Variable	t-Value
A	10.227	A	-2.522	A	2.739
B	35.921	B	0.217	B	-17.281
C	19.310	C	1.833	C	3.232
D	9.083	D	2.739	D	44.158
		E	-0.079		
		F	-0.433		
		G	-7.586		
		H	-1.103		

At the 5 percent significance level, the t-value must exceed 3.182 (for three degrees of freedom) to cause rejection of the hypothesis that a given variable has no effect on the test responses. Thus, in design 1, the effects of A, B, C, and D are indicated important; in design 2, variable G; and in design 3, variables B, C, and D.

Another means of comparing the effects obtained for all system and dummy variables of the three designs involves the term $\frac{|\text{effect on } P_{20}|}{\text{mean value of } P_{20}} \times 100$. For each variable within a given design, the average of P_{20}'s with the variable always at its high level and the average of P_{20}'s with the variable always at its low level (-) are equidistant from the average value of P_{20} for all tests within the design. Thus, $\frac{|\text{effect on } P_{20}|}{\text{mean value of } P_{20}} \times 100$ indicates the change in

percent from the average value of P_{20} caused by testing a variable at its high and low levels. Values of $\dfrac{|\text{effect on } P_{20}|}{\text{mean value of } P_{20}} \times 100$ are presented in Table 2.7:

TABLE 2.7

VALUE OF $\dfrac{|\text{EFFECT ON } P_{20}|}{\text{AVERAGE VALUE OF } P_{20}} \times 100$ FOR EACH VARIABLE OF

THREE PLACKETT-BURMAN DESIGNS

	Plackett-Burman Design Number					
	1		2		3	
Variable	$\dfrac{\text{Effect on } P_{20}}{\text{Average Value of } P_{20}} \times 100$	Variable	$\dfrac{\text{Effect on } P_{20}}{\text{Average Value of } P_{20}} \times 100$	Variable	$\dfrac{\text{Effect on } P_{20}}{\text{Average Value of } P_{20}} \times 100$
A	19.8	A	6.3	A	5.9
B	69.7	B	0.5	B	37.0
C	37.5	C	4.4	C	6.9
D	17.6	D	6.6	D	94.6
(E)	13.1	E	0.2	(E)	21.5
(F)	0.9	F	1.0	(F)	9.8
(G)	6.6	G	18.2	(G)	12.1
		H	2.7		
		(I)	4.1		
		(J)	0.9		
		(K)	0.1		
Average value of P_{20} = 2616 N		Average value of P_{20} = 2111 N		Average value of P_{20} = 2368 N	

These values indicate that the most important system variables are B, C, A, and D from design 1; G from design 2; and D and B from design 3. Taken together, analyses based on data in Figure 2.3 and in Tables 2.6 and 2.7 indicate that the system variables that influence P_{20} most are B, C, A, and D from design 1; G from design 2; and D and B from design 3.

Findings in the above paragraph must be interpreted relative to the present state of knowledge of their influence on track pull performance in sand. The variables indicated most important are index of soil strength G , load W , track width b , track length ℓ (from design 1); minimum pressure in road bogies (from design 2); road-wheel

spacing, track-frame trim angle θ, and possibly location of the drive sprocket (from design 3). It is common practice to keep road-wheel spacing as small as practicable. Track-frame trim angle θ was mechanically controlled during the tests of Plackett-Burman design 3; this can be done in the laboratory, but not in the field. Control over θ in field operations is generally limited to design of the tracked vehicle's weight distribution, i.e. its at-rest center of gravity (or RCG). Turnage [1970] has demonstrated that locating the RCG from 0 to 0.3 ℓ forward of the geometric center-line influences P_{20} negligibly; values of P_{20} are reduced by moving the RCG toward the rear. Values of P_{20} were increased by locating the drive sprocket at the rear (positive effect of parameter C, design 3 in Table 2.5); most present-day tracked vehicles are rear-sprocket driven. Finally, values of P_{20} were increased by decreasing pressure in the road bogies (negative effect of parameter G, design 2). This result agrees with findings of practically all soil-track investigators, in that it suggests track pull improves when load is transferred to the soil with a minimum of load concentration. Thus, it can be recommended that (a) road-wheel spacing be minimized, (b) θ be kept small by locating the track's RCG at or forward of the track's geometric center, (c) the track's drive sprocket be located at the rear, and (d) road bogies be designed as flexible as practicable. Attention will be given the relations among P_{20}, G, b, ℓ, and W in the following chapters, because these relations are not well defined by existing test results. In all tests whose data are examined in the remainder of this

thesis, values of independent variables other than G , b , ℓ , and W were maintained constant at the following levels:

Angle of approach, α	22 deg
Angle of departure, β	22 deg
Road-wheel diameter	17.8 cm
Road-wheel spacing	20.3 cm
Pressure in road bogies	276 kN/m^2
Distribution of pressure in bogies	Uniform
Location of drive sprocket	Rear
Track-shoe height	2.54 cm
Track-shoe thickness	0.32 cm
Track-shoe spacing	3.0 cm
Index of track-belt tension	6890 kN/m^2
Translational velocity	0.6 m/sec
Track-frame trim angle	Unrestrained

CHAPTER III

DEVELOPMENT OF TECHNIQUE TO PREDICT TRACK PULL

A. Determination of basic
    dimensionless terms

Principles from similitude as described by Murphy [1950] are very

useful in developing relations that permit reliable prediction of proto-

type behavior from observations of model performance. The application

of these principles to the sand-track system is described briefly and

informally in the following paragraphs; to obtain a rigorous descrip-

tion of the principles of similitude, the reader is referred to

Murphy [1950].

A basic part of a similitude study is application of dimensional

analysis to the system of concern. The first step is to determine the

pertinent variables. This was done in Chapter II for the system in-

vestigated in this thesis. Independent variables G , b , ℓ , and

W (Table 2.1) were indicated to be deserving of initial study, and they

are referred to hereafter as the "basic variables." The next step is to

determine the number of dimensionless and independent terms required to

express a relation among the variables of interest. These are called

Pi terms after the Buckingham Pi Theorem, which states that the number

of Pi terms (s) equals the total number of variables involved (n) minus

the number of basic dimensions of the variables (b), i.e. $s = n - b$.

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For the soil-track system being studied, $n = 5$ (G , b , ℓ , W , and P_{20} ; the dependent variable P_{20} is included in the total). Basic dimensions are units of the variables in terms of mass, length, and time (MLT units); these are given in Table 3.1 for the variables of interest. Only two basic dimensions, M and L , are included among

TABLE 3.1

BASIC DIMENSIONS OF FIVE TRACK-SAND VARIABLES

Variable	Basic Dimensions (In Units of Mass-Length-Time)
P_{20}	M
G	M/L^3
b	L
ℓ	L
W	M

the five variables, so $s = n - b = 5 - 2 = 3$. The relation among the variables of Table 3.1 can be expressed as $\pi_1 = F(\pi_2, \pi_3)$, where F indicates only that π_1 (which includes the dependent variable) is functionally related to π_2 and π_3 . Development of Pi terms from the variables of Table 3.1 is simplified because P_{20} customarily is expressed in the dimensionless ratio pull/load, or $\dfrac{P_{20}}{W}$, and purely linear terms can be expressed as ratios of a characteristic track dimension. If track length is selected as the characteristic dimension, $\pi_1 = \dfrac{P_{20}}{W}$ and $\pi_2 = \dfrac{b}{\ell}$. The third Pi term must include G and W , so

that the mass dimension M can be balanced. Again with track length ℓ as the characteristic track dimension, $\pi_3 = \dfrac{G\ell^3}{W}$. Thus, the functional relation among P_{20}, G, b, ℓ, and W can be expressed as $\dfrac{P_{20}}{W} = F\left(\dfrac{b}{\ell}, \dfrac{G\ell^3}{W}\right)$.

B. Development of the basic-
    variable prediction term

The form of the functional relation $\dfrac{P_{20}}{W} = F\left(\dfrac{b}{\ell}, \dfrac{G\ell^3}{W}\right)$ can be determined only by analysis of data from tests in which values of dependent Pi term $\dfrac{P_{20}}{W}$ are monitored while values of $\dfrac{b}{\ell}$ and $\dfrac{G\ell^3}{W}$ are varied. The upper plot in Figure 3.1 shows how values of $\dfrac{P_{20}}{W}$ responded to changes in the value of $\dfrac{G\ell^3}{W}$ for $\dfrac{b}{\ell} = \dfrac{1}{4}$. Data for the two sizes of tracks represented (15.2 cm $\times$ 61.0 cm and 30.5 cm $\times$ 121.9 cm) may be described by one central relation, indicating that $\dfrac{G\ell^3}{W}$ is related to $\dfrac{P_{20}}{W}$ in similar fashion for model and prototype. The lower plot in Figure 3.1 presents the $\dfrac{P_{20}}{W}$ versus $\dfrac{G\ell^3}{W}$ relation for four values of $\dfrac{b}{\ell}$ $\left(\dfrac{1}{1}, \dfrac{1}{2}, \dfrac{1}{4}, \text{ and } \dfrac{1}{8}\right)$. (Full lines in both plots of Figure 3.1 are visual lines of best fit.)

To incorporate the influence of $\dfrac{b}{\ell}$ in the prediction term being sought, the relation of $\dfrac{b}{\ell}$ to $\dfrac{G\ell^3}{W}$ was needed. In the logarithmic plot of Figure 3.2, the inverse of $\dfrac{b}{\ell}$ is related to $\dfrac{G\ell^3}{W}$ by the expression

$$\frac{\ell}{b} = K\left(\frac{G\ell^3}{W}\right)^{2/3} \qquad\qquad [3.B.1]$$

where K is a constant of proportionality. Raising both sides of equation [3.B.1] to the 3/2 power yields

RELATION OF $\frac{P_{20}}{W}$ TO $\frac{G\ell^3}{W}$ FOR FOUR VALUES OF $\frac{b}{\ell}$

a. $\frac{b}{\ell} = \frac{1}{4}$

b. $\frac{b}{\ell} = \frac{1}{1}, \frac{1}{2}, \frac{1}{4}$, AND $\frac{1}{8}$
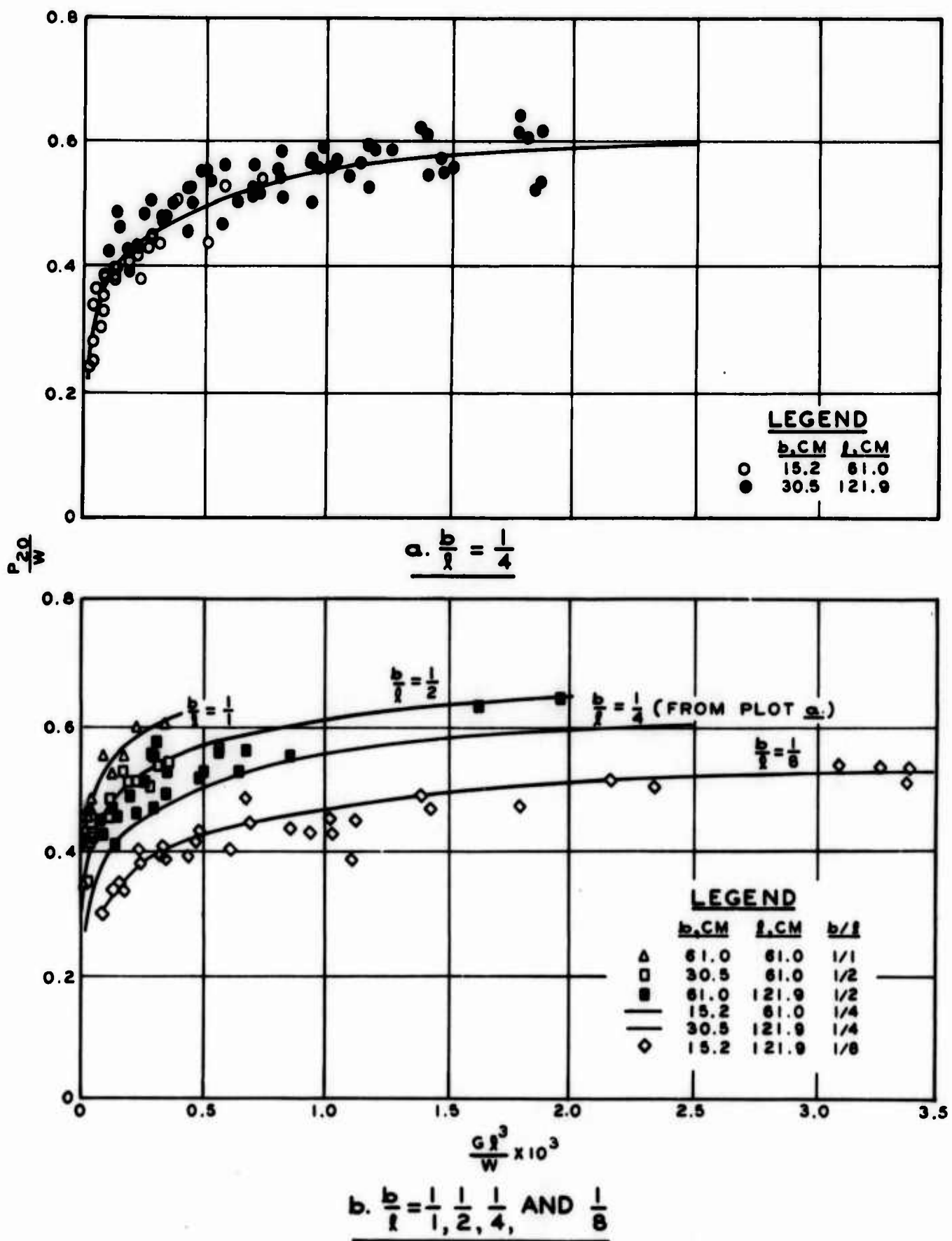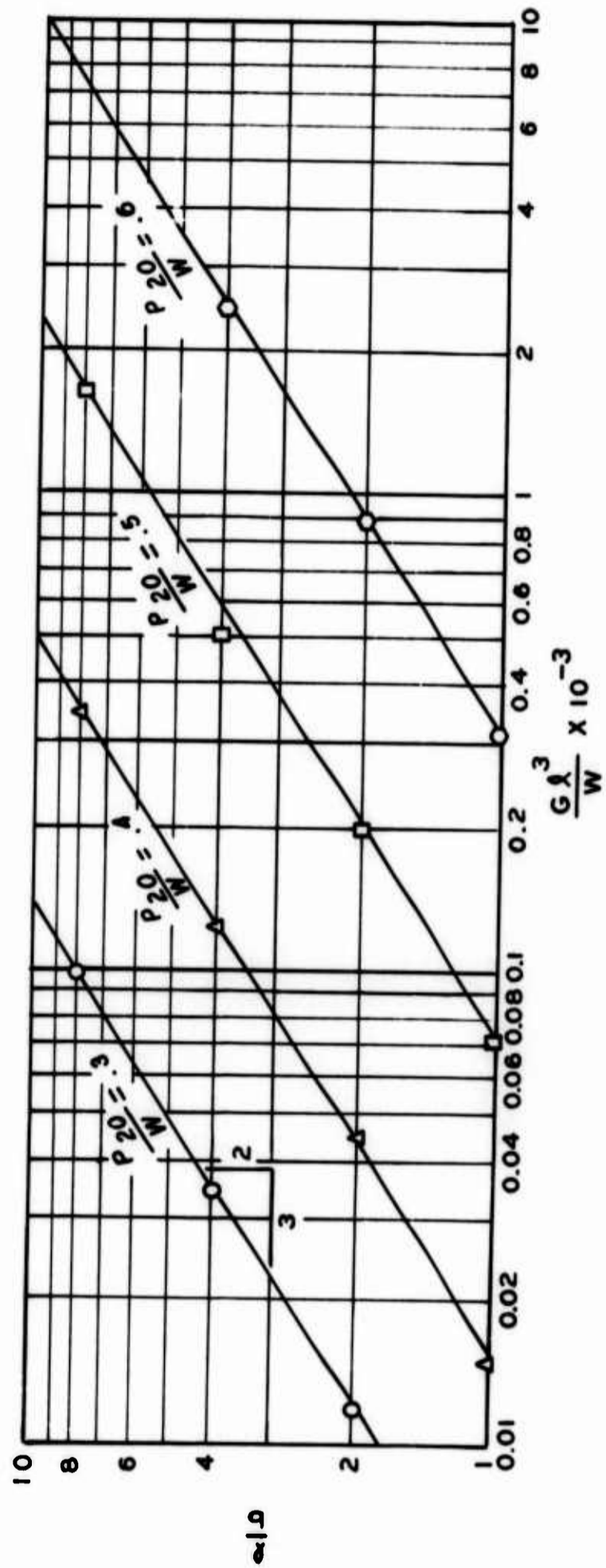
FIGURE 3.1

FIGURE 3.2

NOTE: PLOTTED POINTS WERE OBTAINED
FROM THE FULL LINES OF FIGURE 3.1.

$$\frac{\ell^{3/2}}{b^{3/2}} = K^{3/2} \times \frac{G\ell^3}{W} \ ,$$

or

$$\frac{G}{W} \times (b\ell)^{3/2} = K^{-3/2} \ . \qquad\qquad [3.B.2]$$

The value of $K^{-3/2}$ changes as a function of $\frac{P_{20}}{W}$. Equation [3.B.2] states, then, that for each particular value of $\frac{G(b\ell)^{3/2}}{W}$, there is a corresponding value of $\frac{P_{20}}{W}$. $\frac{G(b\ell)^{3/2}}{W}$ is the dimensionless basic-variable prediction term indicated by relations among $\frac{P_{20}}{W}$, $\frac{b}{\ell}$, and $\frac{G\ell^3}{W}$, to be able to describe $\frac{P_{20}}{W}$ in terms of the four independent track-sand variables G , b , ℓ , and W .

Dimensional analysis has proved valuable in determining the form of a dimensionless, basic-variable prediction term. This technique does not ascertain the functional relation between the prediction term and the dimensionless performance term (or terms) it predicts. Methods of statistics will be applied in the following chapter to define such a relation, and to examine in detail the adequacy of $\frac{G(b\ell)^{3/2}}{W}$ in predicting track pull at 20 percent slip.

CHAPTER IV

CRITICAL EXAMINATION OF BASIC-VARIABLE PREDICTION TERM

A. Differences between prediction equations for
   various levels of the independent variables

In Chapter III, it was determined that there exists a functional relation between $\frac{P_{20}}{W}$ and $\frac{G(b\ell)^{3/2}}{W}$, i.e.

$$\frac{P_{20}}{W} = f\left[\frac{G(b\ell)^{3/2}}{W}\right] . \qquad [4.A.1]$$

A data plot of the relation between $\frac{P_{20}}{W}$ and $\frac{G(b\ell)^{3/2}}{W}$ is presented in arithmetic form in Figure 4.1. (Test data used in the analysis of Section B of Chapter III and in the remaining analyses of this thesis are presented in Table 4.1.) The relation of Figure 4.1 takes near-linear form when the abscissa scale is changed from arithmetic to logarithmic (Figure 4.2). (In Figure 4.2 and hereafter, log represents $\log_{10}$.) The equation of a straight line in Figure 4.2 is of the form

$$\frac{\hat{P}_{20}}{W} = a_0 + a_1 \log \frac{G(b\ell)^{3/2}}{W} \qquad [4.A.2]$$

where

a_0 = the value of $\frac{\hat{P}_{20}}{W}$ at log $\frac{G(b\ell)^{3/2}}{W} = 1.0$ (i.e. the intercept)

a_1 = the slope of the line

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FIGURE 4.1

TABLE 4.1

TEST DATA ANALYZED IN THE DEVELOPMENT OF PREDICTION TERM $\frac{G(b\ell)^{3/2}}{W}$ AND IN THE LINEAR REGRESSION OF $\frac{P_{20}}{W}$ ON LOG $\frac{G(b\ell)^{3/2}}{W}$

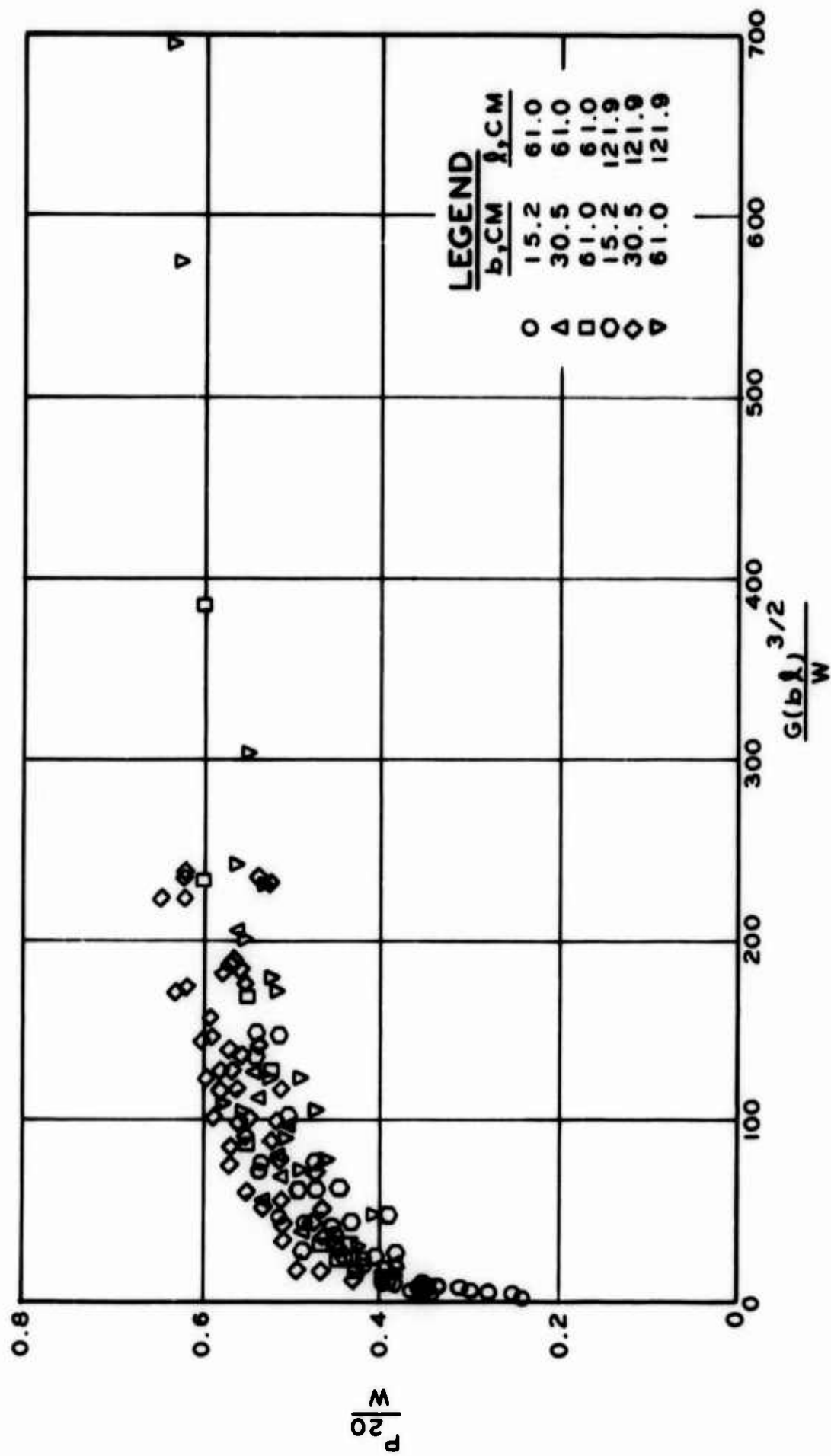| Test No.	G, MN/m³	W, N	I, N	$\frac{I}{W}$	$\frac{G\ell^3}{W}$	$\frac{G(b\ell)^{3/2}}{W}$	Log $\frac{G(b\ell)^{3/2}}{W}$	Test No.	G, MN/m³	W, N	I, N	$\frac{I}{W}$	$\frac{G\ell^3}{W}$	$\frac{G(b\ell)^{3/2}}{W}$	Log $\frac{G(b\ell)^{3/2}}{W}$
			15.2-cm × 61.0-cm Track								15.2-cm × 121.9-cm Track (Continued)					
A-68-0023-1	2.58	2,718	1,143	0.421	215	26.8	1.42813	D-69-0168-1	4.39	7,110	2,749	0.387	1110	48.9	1.68931
24	0.18	2,744	1,219	0.444	511	63.6	1.80346	170	5.75	4,432	2,188	0.502	2350	103	2.01284
34	2.20	2,718	1,041	0.383	230	28.7	1.45788	171	5.92	3,166	1,600	0.535	3390	149	2.17319
35	2.01	6,760	2,598	0.384	97.4	12.1	1.08279	A-69-0172-1	476	13,242	6,437	0.485	472	28.6	1.45637
56	1.20	6,555	2,249	0.341	40.9	5.09	0.70672				30.5-cm × 121.9-cm Track				
57	2.53	6,557	2,500	0.381	87.6	10.9	1.03743	A-67-0034-1	4.23	6,784	3,870	0.571	1130	141	2.14922
58	1.98	6,770	2,474	0.365	53.0	6.59	0.81889	35	4.72	6,761	4,012	0.593	1240	158	2.19866
59	4.42	6,961	2,771	0.398	141	17.5	1.24304	36	4.36	4,439	2,891	0.651	1780	223	2.34830
D-68-0018-1	1.15	3,114	1,103	0.354	83.8	10.4	1.01703	37	4.42	4,404	2,713	0.616	1820	238	2.37658
187	1.19	2,455	909	0.365	115	14.3	1.15534	38	4.51	4,586	2,847	0.621	1780	223	2.34830
188	1.08	12,093	2,910	0.241	20.3	2.52	0.40140	39	4.75	4,573	2,847	0.623	1880	235	2.37107
189	2.21	12,218	3,043	0.249	41.1	5.11	0.70842	41	4.41	7,304	4,008	0.549	1090	137	2.13672
190	2.25	4,140	1,584	0.383	123	15.3	1.18469	42	5.17	6,646	3,661	0.551	1410	176	2.24551
191	2.09	2,472	1,015	0.411	192	23.9	1.37840	43	3.04	6,744	3,470	0.515	817	102	2.00860
192	2.25	1,909	900	0.509	383	47.7	1.67852	44	2.59	6,494	3,381	0.521	722	90.4	1.95617
193	2.55	4,821	1,904	0.395	120	14.9	1.17319	45	3.47	6,663	3,376	0.507	943	118	2.07128
194	2.53	11,955	3,375	0.282	48.0	6.00	0.77815	46	2.91	4,448	2,624	0.590	1190	148	2.17026
195	4.84	1,492	799	0.535	984	72.7	1.86153	47	2.57	4,506	2,598	0.577	1030	129	2.11059
196	3.85	3,144	1,413	0.449	278	34.6	1.53908	A-68-0001-1	2.57	6,686	3,447	0.516	696	87.1	1.94002
197	4.75	11,970	4,975	0.407	71.1	8.84	0.94645	6	3.47	4,586	2,882	0.628	1370	172	2.23553
198	4.16	11,860	4,010	0.340	79.6	9.93	0.99695	7	3.51	4,542	2,802	0.617	1400	175	2.24304
199	4.13	3,446	1,548	0.445	272	33.8	1.52892	8	3.57	6,752	3,781	0.560	958	120	2.07918
200	4.21	1,291	705	0.546	740	92.1	1.96426	9	1.12	6,383	3,025	0.474	321	40.2	1.60423
D-70-0026-1	5.71	6,990	2,834	0.408	186	23.2	1.36549	10	1.91	6,646	3,585	0.539	521	65.2	1.81425
29	5.72	4,861	2,110	0.434	267	33.2	1.52114	11	1.17	6,668	3,229	0.484	318	39.8	1.59988
40	5.41	5,903	1,700	0.427	309	38.4	1.58433	12	1.02	4,323	2,291	0.530	427	53.5	1.72835
			30.5-cm × 61.0-cm Track					13	2.23	4,346	2,482	0.571	929	116	2.06446
D-69-0150-1	1.31	3,595	1,613	0.449	102	35.9	1.55509	14	1.08	4,453	2,260	0.507	439	55.0	1.74036
181	1.63	7,469	3,111	0.417	49.5	17.5	1.24304	17	1.95	4,413	2,460	0.557	800	100	2.00000
182	1.68	14,185	4,948	0.349	26.9	9.50	0.97772	18	2.24	6,489	3,305	0.509	625	78.3	1.89376
184	4.18	13,873	5,960	0.429	68.4	24.2	1.38382	35	2.71	11,583	6,152	0.531	424	53.0	1.72428
194	4.14	8,005	3,883	0.485	117	41.5	1.61805	A-68-0036-1	2.60	9,501	5,276	0.555	496	62.0	1.79239
185	4.19	3,438	1,725	0.502	277	97.8	1.99034	37	4.63	8,977	5,276	0.576	934	117	2.06819
201	4.56	1,799	1,009	0.560	576	204	2.30963	38	1.77	8,954	4,528	0.506	358	44.8	1.65128
202	4.58	2,866	1,544	0.539	363	128	2.10721	39	1.20	8,781	4,275	0.487	248	31.0	1.49136
203	4.65	5,428	2,763	0.509	194	68.7	1.83696	40	3.37	8,736	4,951	0.567	699	87.5	1.94201
204	3.14	4,400	2,318	0.527	162	57.3	1.75815	52	4.35	6,792	4,083	0.601	1160	145	2.16137
205	3.10	3,088	1,570	0.508	228	80.6	1.90634	53	3.70	6,828	4,057	0.594	982	123	2.08991
206	3.68	2,181	1,169	0.535	321	113	2.05308	D-68-0168-1	1.26	22,970	9,815	0.427	99.4	12.4	1.09342
D-70-0002-1	1.02	3,903	1,623	0.416	59.3	21.0	1.32222	169	1.25	15,930	7,395	0.467	142	17.8	1.25042
26	4.89	13,847	6,035	0.436	97.4	28.3	1.45179	170	1.28	12,213	5,160	0.423	190	23.8	1.37658
			61.0-cm × 61.0-cm Track					171	2.69	11,750	5,398	0.459	415	51.9	1.71517
D-70-0012-1	0.98	8,709	3,937	0.449	25.4	25.4	1.40483	172	2.75	15,050	7,280	0.484	331	41.4	1.61700
13	0.97	11,600	4,982	0.429	19.0	19.0	1.27875	173	2.79	23,215	10,000	0.431	218	27.2	1.43457
14	1.05	23,104	7,937	0.344	10.3	10.3	1.01284	D-69-0155-1	5.48	17,426	8,260	0.473	570	71.3	1.85309
15	2.27	14,143	6,601	0.467	36.4	36.4	1.56110	156	5.28	11,790	6,906	0.587	811	102	2.00860
16	2.32	11,556	5,552	0.480	45.6	45.6	1.65896	157	5.46	9,670	5,490	0.567	1020	128	2.10721
17	2.19	5,655	3,111	0.550	87.9	87.9	1.94399	158	6.44	9,972	5,292	0.531	1170	146	2.16435
18	5.37	9,468	4,951	0.523	129	129	2.11059	159	6.40	7,914	4,572	0.579	1460	183	2.26245
19	5.41	7,228	3,981	0.551	170	170	2.23045	160	6.53	8,040	4,446	0.555	1470	184	2.26482
20	5.03	4,968	2,990	0.596	233	233	2.36749	161	6.01	7,230	4,084	0.565	1510	188	2.27416
21	1.99	13,759	6,352	0.468	33.3	33.3	1.52244	162	5.78	5,660	2,944	0.524	1850	232	2.36549
117	6.23	5,570	2,200	0.601	385	385	2.58546	163	5.36	5,158	2,774	0.538	1880	236	2.37291
			15.2-cm × 121.9-cm Track					164	2.00	13,199	6,710	0.508	268	34.4	1.53656
A-68-0022-1	4.40	4,519	2,131	0.472	1790	78.7	1.89597	190	0.94	9,239	3,618	0.392	185	23.1	1.36361
15	3.55	4,528	2,120	0.467	1420	62.5	1.79588	191	1.06	13,849	6,773	0.489	139	17.4	1.24055
27	2.51	4,657	2,113	0.454	1020	44.7	1.65031	192	4.53	13,765	7,818	0.568	576	74.6	1.87274
28	1.28	4,724	2,051	0.434	491	21.6	1.33445	193	4.36	9,511	5,197	0.546	813	104	2.01703
29	1.87	4,584	2,166	0.443	694	30.5	1.48430	194	3.39	9,910	5,248	0.530	695	77.5	1.88930
30	1.10	6,801	2,678	0.394	317	14.0	1.14613	195	3.46	12,375	6,874	0.555	477	63.4	1.80209
31	1.25	7,280	2,865	0.393	435	19.2	1.28330				61.0-cm × 121.9-cm Track				
32	2.46	7,236	2,918	0.403	613	27.0	1.43136	D-68-0149-1	6.05	16,070	9,000	0.560	682	241	2.38202
33	4.27	7,099	3,025	0.429	942	41.5	1.61805	154	1.13	9,168	4,170	0.457	224	79.5	1.90037
34	4.17	7,299	3,140	0.430	1040	45.6	1.65896	155	1.09	23,080	10,430	0.452	85.5	30.3	1.48144
42	1.78	2,313	1,140	0.489	1390	61.4	1.78817	157	1.49	9,018	4,225	0.469	299	106	2.02531
43	2.79	2,318	1,201	0.518	2140	95.0	1.97772	158	1.57	23,215	10,770	0.464	123	43.4	1.63749
44	3.92	2,322	1,250	0.538	3100	139	2.13354	160	2.61	9,318	4,865	0.522	507	180	2.25527
45	4.37	2,344	1,192	0.509	3380	149	2.17319	161	2.61	23,010	11,215	0.487	205	72.7	1.86153
46	1.19	8,421	3,559	0.403	238	10.5	1.02119	163	3.30	9,148	4,818	0.527	653	231	2.36361
47	1.06	6,141	2,701	0.405	429	14.5	1.16137	164	3.26	22,815	11,620	0.509	259	91.6	1.96190
48	1.19	4,509	1,904	0.414	469	20.7	1.31597	166	4.38	9,248	5,085	0.550	858	304	2.48287
49	5.27	6,877	3,020	0.439	861	37.9	1.57864	167	4.47	23,220	12,265	0.528	349	123	2.08991
50	4.84	7,042	3,145	0.447	1120	49.2	1.69197	D-69-0124-1	1.14	6,898	3,822	0.554	299	106	2.02531
51	4.04	2,237	1,189	0.535	3270	144	2.15830	125	1.04	6,026	3,472	0.576	313	111	2.04532
174	2.33	23,020	7,755	0.337	183	8.07	0.90687	126	0.90	5,592	3,114	0.557	292	103	2.01284
175	2.15	15,835	6,015	0.380	246	10.8	1.03342	128	1.07	13,616	5,580	0.409	142	50.4	1.70243
176	2.25	12,123	4,703	0.388	335	14.8	1.17026	137	1.19	6,150	3,020	0.491	350	124	2.09342
177	1.00	11,903	4,180	0.351	152	6.70	0.82607	140	1.22	15,708	7,134	0.454	141	49.8	1.69723
178	1.14	16,000	5,400	0.338	129	5.68	0.75435	145	1.21	25,162	10,666	0.424	87.1	30.8	1.48855
179	1.18	23,250	6,625	0.288	91.9	4.05	0.60746	149	2.08	7,738	3,986	0.515	487	172	2.23553
								153	2.16	6,892	3,784	0.554	568	201	2.30320
								D-70-0114-1	6.82	7,508	4,767	0.627	1620	576	2.76042
								115	6.79	6,257	4,011	0.641	1960	696	2.84261

31

RELATION OF $\frac{P_{20}}{W}$ TO LOG $\frac{G(b\,\ell)^{3/2}}{W}$

$\frac{\widehat{P_{20}}}{W} = 0.204482 + 0.181470 \, LOG \, \frac{G(b\,\ell)^{3/2}}{W}$

95 PERCENT CONFIDENCE LIMITS ON $\frac{P_{20}}{W}$

LEGEND

b,CM	ℓ,CM
15.2	61.0
30.5	61.0
61.0	61.0
15.2	121.9
30.5	121.9
61.0	121.9

FIGURE 4.2

Values of a_0 and a_1 were estimated from the linear equation that minimized the sum of squared deviations of the ordinate variable about the regression line. The following least-squares equation resulted:

$$\frac{\widehat{P}_{20}}{W} = 0.204482 + 0.161470 \; \log \cdot \frac{G(b\ell)^{3/2}}{W} \qquad [4.A.3]$$

This equation is represented as a continuous line in Figure 4.2.

Dashed lines in Figure 4.2 define 95 percent confidence limits for the value of $\frac{P_{20}}{W}$ that would be predicted by the regression equation for a particular observed value of $\log \frac{G(b\ell)^{3/2}}{W}$. From Davies [1961], these limits are given by

$$\widehat{Y}_0 \pm t_{0.025(n-2)} \; s \; \sqrt{1 + \frac{1}{n} + \frac{(X_0 - \overline{X})^2}{\Sigma(X_i - \overline{X})^2}} \qquad [4.A.4]$$

where

$\widehat{Y}_0$ = predicted value of $\frac{P_{20}}{W}$ for $X = X_0$

s = standard error of estimate of Y on X

n = number of data points

$X_i = \log \frac{G(b\ell)^{3/2}}{W}$ for the ith data point included in the linear regression

X_0 = a future observation of $\log \frac{G(b\ell)^{3/2}}{W}$

Information from the first line of Table 4.2 was substituted in equation [4.A.4], and values 0.068, 0.066, 0.065, and 0.067 were obtained for

$$\pm t_{0.025(n-2)} \; s \; \sqrt{1 + \frac{1}{n} + \frac{(X_0 - \overline{X})^2}{\Sigma(X - \overline{X})^2}} \quad \text{for} \quad X_0 \text{ values of 0, 1, 2, and 3,}$$

respectively. Thus, the separation between the dashed lines of Figure 4.2 increases as the difference between values of X_0 and $\overline{X}$ increases, but the size of this increase is practically imperceptible for the range

TABLE 4.2

LINEAR REGRESSION OF $\frac{P_{20}}{W}$ ON LOG $\frac{G(b\ell)^{3/2}}{W}$ FOR ALL TEST DATA AND FOR TEST

DATA SEPARATED BY LEVELS OF $b \times \ell$, G, AND W

Sub-group No.	No. Data Points	Values of Variables Common Within Subgroup (b, cm ℓ, cm; G; W)	Sum of Squares			Standard Error of Estimate s	Standard Deviation of X, s_x	Standard Deviation of Y, s_y	Mean of X	Mean of Y	$\frac{P_{20}}{W} = a_0 + a_1 \log \frac{G(b\ell)^{3/2}}{W}$		Coefficient of Correlation, r	ΣX_i^2
			About the Mean $\Sigma(Y_1-\bar{Y})^2$	About Regression $\Sigma(Y_1-Y_{est})^2$	Due to Regression $\Sigma(Y_{est}-\bar{Y})^2$						a_0	a_1		
1	158	All test data	1.087021	0.171459	0.915562	0.033153	0.472936	0.083209	1.72367	0.482804	0.204482	0.161470	0.917751	504.5410
		b, cm ℓ, cm												
2	26	15.2 61.0	0.143505	0.018452	0.125053	0.027728	0.392087	0.075764	1.24460	0.393115	0.168611	0.180383	0.933500	44.1181
3	14	30.5 61.0	0.047724	0.003135	0.044589	0.016164	0.379108	0.060590	1.67998	0.475786	0.216304	0.154482	0.966593	41.3672
4	11	61.0 61.0	0.059378	0.003587	0.055791	0.019963	0.494301	0.077057	1.78806	0.496182	0.225989	0.151109	0.969328	37.6121
5	30	15.2 121.9	0.115093	0.014902	0.100191	0.023070	0.446187	0.062998	1.49765	0.433900	0.236609	0.131734	0.933019	73.0618
6	55	30.5 121.9	0.169958	0.054387	0.114971	0.032034	0.326285	0.056002	1.93122	0.538164	0.265057	0.141416	0.823932	210.8784
7	22	61.0 121.9	0.077243	0.018788	0.058454	0.030650	0.363933	0.066048	2.074499	0.514864	0.214054	0.144970	0.869920	97.5039
		G, MN/m³												
8	49	0.90 ≤ G < 2 MN/m³	0.240599	0.047890	0.192709	0.031921	0.405150	0.070799	1.40250	0.438020	0.218861	0.156392	0.894962	104.2617
9	34	2 ≤ G < 3 MN/m³	0.236676	0.028789	0.207888	0.029994	0.441022	0.084688	1.59767	0.459147	0.171617	0.179969	0.937210	93.2048
10	21	3 ≤ G < 4 MN/m³	0.100793	0.025119	0.075674	0.036360	0.308499	0.070990	1.89903	0.516333	0.137686	0.199390	0.866480	77.6357
11	31	4 ≤ G < 5 MN/m³	0.180623	0.038303	0.142320	0.036343	0.371804	0.077594	1.92770	0.517323	0.162215	0.185250	0.897660	119.3445
12	14	5 ≤ G < 6 MN/m³	0.044124	0.011296	0.032828	0.030681	0.327348	0.058259	2.02253	0.516143	0.205662	0.153511	0.862550	58.6618
13	9	6 ≤ G < 7 MN/m³	0.026998	0.002006	0.024992	0.016927	0.319453	0.058093	2.37151	0.567000	0.152068	0.174965	0.952139	51.4330
		W												
14	16	0 ≤ W < 3 kN	0.055897	0.010260	0.045637	0.027071	0.336884	0.061045	1.83911	0.491062	0.189888	0.163761	0.903575	55.8191
15	39	3 ≤ W < 6 kN	0.230384	0.034355	0.196029	0.030472	0.403518	0.077864	1.87061	0.512487	0.179531	0.177994	0.922431	142.6551
16	48	6 ≤ W < 9 kN	0.288770	0.043612	0.245158	0.030791	0.476084	0.078384	1.79322	0.495437	0.223403	0.151702	0.921397	165.0038
17	22	9 ≤ W < 12 kN	0.170353	0.033014	0.137339	0.040629	0.514518	0.090067	1.70726	0.469545	0.201205	0.157176	0.897888	59.6835
18	15	12 ≤ W < 15 kN	0.127393	0.022674	0.104719	0.041763	0.398753	0.095391	1.34378	0.430933	0.139478	0.216893	0.906652	29.3121
19	7	15 ≤ W < 18 kN	0.032197	0.004274	0.027923	0.029236	0.546212	0.073254	1.51280	0.451857	0.262916	0.124895	0.931271	17.8099
20	0	18 ≤ W < 21 kN	---	---	---	---	---	---	---	---	---	---	---	---
21	10	21 ≤ W < 24 kN	0.054220	0.003331	0.050889	0.020406	0.492668	0.077574	1.40912	0.427700	0.212755	0.152539	0.968759	22.0407
22	1	24 ≤ W < 27 kN	---	---	---	---	---	---	---	---	---	---	---	---

of X_0 values considered. The magnitude of this separation indicates that $\dfrac{P_{20}}{W}$ can be predicted with reasonable precision for the full range of values of $\log \dfrac{G(b\ell)^{3/2}}{W}$ that might reasonably be encountered.

The relation in Figure 4.2 is very useful. The pull that a track can develop for a given combination of values of G , W , b , and ℓ is often important (consider, for example, towing another vehicle or pushing a soil mass with an attached blade in front). Also, from a large number of field tests, the WES Soils Division [1951] determined that $(0.90) \times \left(\dfrac{P_{20}}{W} \text{ attainable on level sand} \right)$ very closely matches the maximum slope that a tracked vehicle can negotiate in dry, loose sands.

In a linear regression of form $\hat{Y}_i = a_0 + a_1 X_i + \epsilon_i$, the assumption is made that errors (ϵ_i's) are independent, have zero mean and constant variance σ^2 , and follow a normal distribution. Residuals obtained by subtracting predicted Y_i (i.e. $\hat{Y}_i$) from measured Y_i can be considered to represent the observed errors if the model is correct. To obtain an indication of whether the above assumptions are satisfied, the residuals can be plotted on normal probability paper. In this plot, the abscissa represents the value of the residual; the ordinate value of the mth ordered residual is the $\dfrac{m - \frac{1}{2}}{n} \times 100$ percentile of the standard normal distribution, where n is the total number of residuals and m takes values 1, 2,..., n . Figure 4.3 shows that the data closely approximate a straight line that passes through (0, 50), thus indicating a normal distribution centered on zero. (Placement of the full line was done by causing it to pass through (0, 50) and (s, 84), where s = 0.033153 , from Table 4.2.) The relation of Figure 4.3 does

NORMAL PROBABILITY PLOT OF RESIDUALS
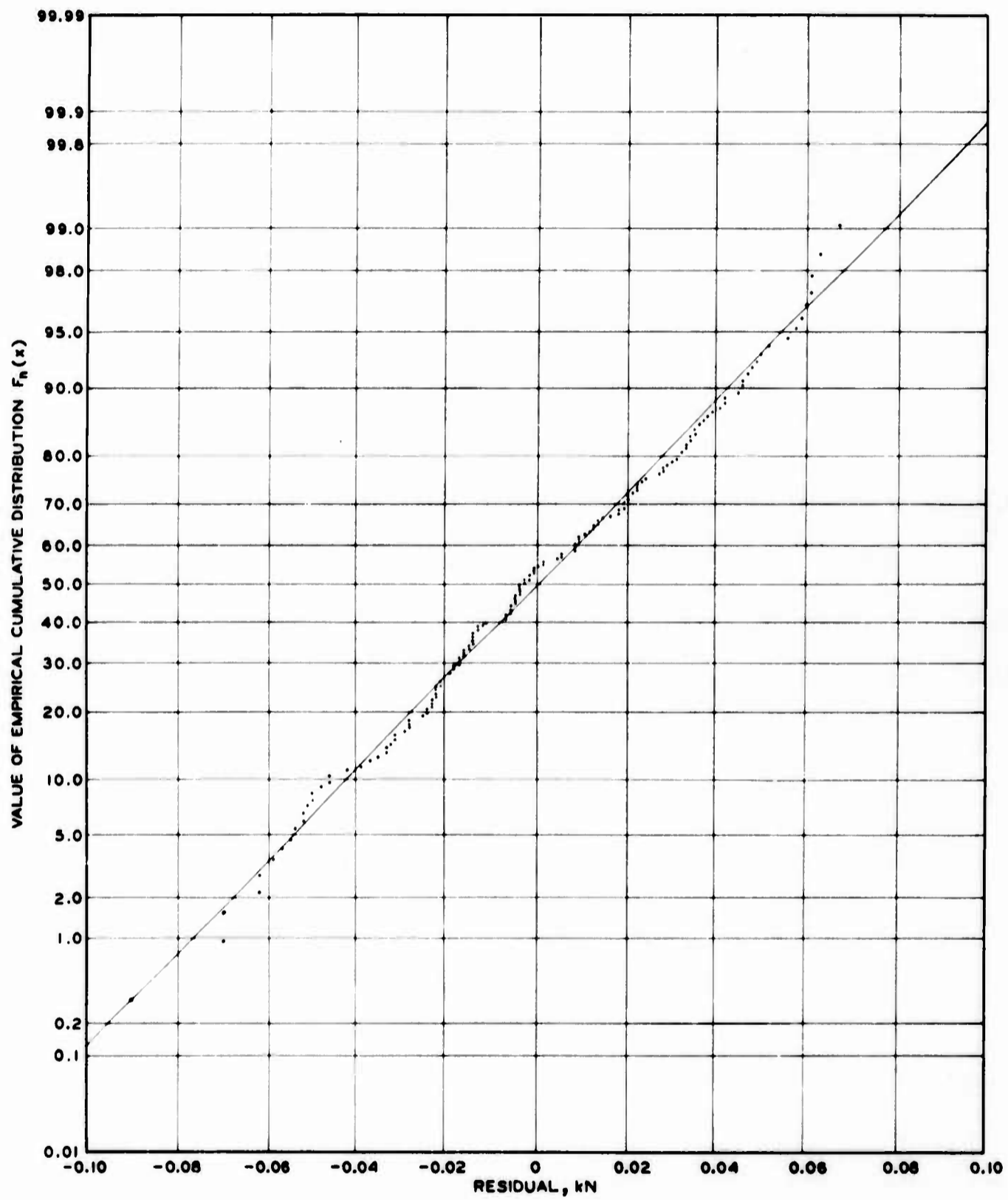FROM APPLICATION OF EQUATION [4.A.3]

FIGURE 4.3

not contradict the assumption that the linear regression model is correct.

An important concern of this chapter is a validation of the results from the dimensional analysis in Chapter III. One means of evaluating whether the influence on $\frac{P_{20}}{W}$ of each variable included in $\frac{G(b\ell)^{3/2}}{W}$ is adequately accounted for by the form of this term is to compare results of linear regressions of $\frac{P_{20}}{W}$ on $\log \frac{G(b\ell)^{3/2}}{W}$ for groups of test data separated by levels of values of b , ℓ , G , and W . Accordingly, the test data were segregated according to (a) the six combinations of $b \times \ell$, (b) six levels of G , and (c) nine levels of W . A summary of information obtained in fitting least-squares regression equations of the form $\frac{\hat{P}_{20}}{W} = a_0 + a_1 \log \frac{G(b\ell)^{3/2}}{W}$ to these subgroups of data is given in Table 4.2.

The precision with which a given regression line could be used to predict $\frac{P_{20}}{W}$ for each of the data subgroups is indicated to be quite good by the small values obtained for standard error of estimate s (ranging from 0.0162 to 0.0418). Of more interest than the variation about each least-squares line was the determination of whether values of intercept and slope for each data subgroup differed significantly from values for intercept and slope when all data except that of the data subgroup of interest were considered. From Draper and Smith [1966], the variances of intercept and slope can be estimated by $\dfrac{s^2 \sum x_i^2}{n \sum (X_i - \overline{X})^2}$ and $\dfrac{s^2}{\sum (X_i - \overline{X})^2}$, respectively, and are designated here as $V(a_0)$ and $V(a_1)$. If small letters indicate values using any one subgroup of data,

and capital letters indicate values using all the remaining data, the
hypotheses $E(a_0) = E(A_0)$ and $E(a_1) = E(A_1)$ are tested at the 5 per-
cent significance level by comparing tabled values of $t_{0.975(df)}$
against values of the computed $\underline{t}$-statistics $\underline{t}_0 = \dfrac{|a_0 - A_0|}{\sqrt{V(a_0) + V(A_0)}}$ and

$\underline{t}_1 = \dfrac{|a_1 - A_1|}{\sqrt{V(a_1) + V(A_1)}}$, respectively. Denominators in these two terms

reflect the assumption that the variances of a_0 and A_0 and of a_1
and A_1 are unequal; this seems reasonable from the much smaller num-
ber of data points used to compute a_0 and a_1 than to compute A_0
and A_1 , respectively. From Dixon and Massey [1957], an approxima-
tion to the number of degrees of freedom appropriate for these $\underline{t}$-tests
is defined as

$$df = \frac{\left[V(a_X) + V(A_X) \right]^2}{\dfrac{\left[V(a_X) \right]^2}{n_{a_X} + 1} + \dfrac{\left[V(A_X) \right]^2}{n_{A_X} + 1}} - 2$$

for $\underline{t}_X$, with $X = 0$ and 1 for our considerations.

Results of testing $E(a_0) = E(A_0)$ and $E(a_1) = E(A_1)$ are pre-
sented in Tables 4.3 and 4.4, respectively. In Table 4.3, examination
of the test data caused the hypothesis $E(a_0) = E(A_0)$ to be rejected
in five cases: for $b \times \ell$ values 15.2 cm $\times$ 61.0 cm, 15.2 cm $\times$ 121.9
cm , and 30.5 cm $\times$ 121.9 cm , and for G values 0.90 $MN/m^3 \leq G$
< 2 MN/m^3 and 2 $MN/m^3 \leq G \leq 3$ MN/m^3 . Thus, it appears that for
$\log \dfrac{G(b\ell)^{3/2}}{W} = 1$, predicted values of $\dfrac{P_{20}}{W}$ for the narrowest track

TABLE 4.3

TESTING THE HYPOTHESIS $E(a_0) = E(A_0)$ FOR LINEAR REGRESSIONS $\dfrac{\hat{P}_{20}}{W} = a_0 + a_1 \, LOG \, \dfrac{G(b\ell)^{3/2}}{W}$

AND $\dfrac{\hat{P}_{20}}{W} = A_0 + A_1 \, LOG \, \dfrac{G(b\ell)^{3/2}}{W}$

		Information from $\dfrac{\hat{P}_{20}}{W} = a_0 + a_1 \, LOG \, \dfrac{G(b\ell)^{3/2}}{W}$			Information from $\dfrac{\hat{P}_{20}}{W} = A_0 + A_1 \, Log \, \dfrac{G(b\ell)^{3/2}}{W}$				Testing the Hypothesis $E(a_0) = E(A_0)$			
Sub-group No.	Values of Variables Common Within Subgroup	a_0	n_{a_0}	$v(a_0)$	Subgroup Excluded From Group	A_0	n_{A_0}	$v(A_0)$	$t_0 = \dfrac{\|a_0 - A_0\|}{\sqrt{v(a_0) + v(A_0)}}$	df_{t_0}	$t_{0.975}(df_{t_0})$	Is Computed t_0 Larger Than Tabled $t_{0.975}(df_{t_0})$?
					Group I (All Test Data Separated into Subgroups by Combinations of b, cm $\times \ell$, cm)							
2	15.2×61.0	0.168611	26	0.0003394	2	0.226241	132	0.0000160	3.057	28	2.048	Yes
3	30.5×61.0	0.215304	14	0.0004132	3	0.203792	144	0.0000115	0.607	14	2.145	No
4	61.0×61.0	0.225989	11	0.0005577	4	0.203045	147	0.0000112	0.962	11	2.201	No
5	15.2×121.9	0.236609	30	0.0002245	5	0.203000	128	0.0000142	2.175	34	2.034	Yes
6	30.5×121.9	0.265057	55	0.0006844	6	0.209568	103	0.0000079	2.109	56	2.004	Yes
7	61.0×121.9	0.214054	22	0.0014970	7	0.193199	136	0.0000102	0.537	22	2.074	No
					Group II (All Test Data Separated into Subgroups by Values of G in MN/m^3)							
8	$0.90 \le G < 2$	0.218861	49	0.0002752	8	0.182399	109	0.0000203	2.121	56	2.004	Yes
9	$2 \le G < 3$	0.171617	34	0.0003842	9	0.213978	124	0.0000135	2.124	36	2.029	Yes
10	$3 \le G < 4$	0.137686	21	0.0025675	10	0.206612	137	0.0000101	1.397	21	2.080	No
11	$4 \le G < 5$	0.160215	31	0.0012261	11	0.210426	127	0.0000135	1.426	31	2.040	No
12	$5 \le G < 6$	0.205663	14	0.0028314	12	0.201765	144	0.0000105	0.073	14	2.145	No
13	$6 \le G < 7$	0.152068	9	0.0020056	13	0.199311	149	0.0000113	1.052	9	2.262	No
					Group III (All Test Data Separated into Subgroups by Values of W in kW)							
14	$0 \le W < 3$	0.189888	16	0.0015024	14	0.204753	142	0.0000108	0.382	16	2.120	No
15	$3 \le W < 6$	0.179531	39	0.0005489	15	0.211420	119	0.0000124	1.346	40	2.021	No
16	$6 \le W < 9$	0.223403	48	0.0003126	16	0.197039	110	0.0000150	1.457	52	2.008	No
17	$9 \le W < 12$	0.201405	22	0.0009405	17	0.205064	136	0.0000109	0.125	22	2.074	No
18	$12 \le W < 15$	0.139478	15	0.0016489	18	0.207175	143	0.0000108	1.662	15	2.131	No
19	$15 \le W < 18$	0.262916	7	0.0012149	19	0.206647	151	0.0000108	1.779	7	2.365	No
20	$18 \le W < 21$	--	0	--	20	0.204482	158	0.0000100	--	--	--	--
21	$21 \le W < 24$	0.212755	10	0.0004201	21	0.204498	148	0.0000118	0.397	10	2.228	No
22	$24 \le W < 27$	--	1	--	22	0.204859	157	0.0000101	--	--	--	--

$df_{t_0} = \dfrac{\left[v(a_0) + v(A_0) \right]^2}{\dfrac{[v(a_0)]^2}{n_{a_0} + 1} + \dfrac{[v(A_0)]^2}{n_{A_0} + 1}} - 2$

TABLE 4.4

TESTING THE HYPOTHESIS $E(a_1) = E(A_1)$ FOR LINEAR REGRESSIONS $\frac{\hat{P}_{20}}{W} = a_0 + a_1 \log \frac{G(b\ell)^{3/2}}{W}$

$$\frac{\hat{P}_{20}}{W} = A_0 + A_1 \log \frac{G(b\ell)^{3/2}}{W}$$

AND $\frac{\hat{P}_{20}}{W} = A_0 + A_1 \log \frac{G(b\ell)^{3/2}}{W}$

Sub-group No.	Information from $\frac{\hat{P}_{20}}{W} = a_0 + a_1 \log \frac{G(b\ell)^{3/2}}{W}$ Values of Variables Common Within Subgroup	a_1	n_{a_1}	$V(a_1)$	Information from $\frac{\hat{P}_{20}}{W} = A_0 + A_1 \log \frac{G(b\ell)^{3/2}}{W}$ Subgroup Excluded From Group	A_1	n_{A_1}	$V(A_1)$	$t_1 = \frac{\lvert a_1 - A_1 \rvert}{\sqrt{V(a_1) + V(A_1)}}$	$df \cdot t_1$	$t_{0.975}(df\,t_1)$	Is Computed t_1 Larger Than Tabled $t_{0.975}(df\,t_1)$?
	Group I (All Test Data Separated into Subgroups by Combinations of b, cm × ℓ, cm)											
2	15.2 × 61.0	0.180383	26	0.0002000	2	0.150838	132	0.0000460	1.884	39	2.023	No
3	30.5 × 61.0	0.155482	14	0.0001398	3	0.161865	144	0.0000357	0.557	22	2.074	No
4	61.0 × 61.0	0.151109	11	0.0001631	4	0.162176	147	0.0000354	0.786	16	2.120	No
5	15.2 × 121.9	0.131734	30	0.0000922	5	0.163941	128	0.0000421	2.779	61	2.000	Yes
6	30.5 × 121.9	0.141416	55	0.0001785	6	0.151084	103	0.0000276	0.673	72	1.995	No
7	61.0 × 121.9	0.144970	22	0.0003378	7	0.170633	136	0.0000342	1.331	26	2.056	No
	Group II (All Test Data Separated into Subgroups by Values of G in MN/m³)											
8	0.90 ≤ G < 2	0.156302	49	0.0001293	8	0.171588	109	0.0000553	1.118	93	1.989	No
9	2 ≤ G < 3	0.179969	34	0.0001402	9	0.156586	124	0.0000408	1.738	55	2.005	No
10	3 ≤ G < 4	0.199390	21	0.0006945	10	0.158665	137	0.0000326	1.525	23	2.069	No
11	4 ≤ G < 5	0.185250	31	0.0003185	11	0.157690	127	0.0000355	1.465	38	2.025	No
12	5 ≤ G < 6	0.153511	14	0.0006757	12	0.163930	144	0.0000341	0.391	15	2.131	No
13	6 ≤ G < 7	0.174965	9	0.0003509	13	0.165272	149	0.0000370	0.492	11	2.201	No
	Group III (All Test Data Separated into Subgroups by Values of W in kW)											
14	0 ≤ W < 3	0.163761	16	0.0004307	14	0.161995	142	0.0000343	0.082	18	2.101	No
15	3 ≤ W < 6	0.177994	39	0.0001501	15	0.156164	119	0.0000408	1.580	62	1.999	No
16	6 ≤ W < 9	0.151702	48	0.0000909	16	0.165504	110	0.0000485	1.169	101	1.986	No
17	9 ≤ W < 12	0.157176	22	0.0002599	17	0.162127	136	0.0000342	0.272	27	2.052	No
18	12 ≤ W < 15	0.216693	15	0.0004438	18	0.159380	143	0.0000326	1.943	16	2.120	No
19	15 ≤ W < 18	0.124895	7	0.0004775	19	0.163599	151	0.0000335	1.712	8	2.306	No
20	18 ≤ W < 21	--	0	--	20	0.161470	158	0.0000313	--	--	--	--
21	21 ≤ W < 24	0.152539	10	0.0001906	21	0.161628	148	0.0000360	0.604	14	2.145	No
22	24 ≤ W < 27	--	1	--	22	0.161329	157	0.0000315	--	--	--	--

$$df \cdot t_0 = \frac{[V(a_1) + V(A_1)]^2}{\dfrac{[V(a_1)]^2}{n_{a_1}+1} + \dfrac{[V(A_1)]^2}{n_{A_1}+1}} - 2$$

and for the lowest strength sand test sections may be different from those of the preponderance of the test data; a similar statement might be made for the longest track, but the test data do not support this argument as well.

In Table 4.4 the hypothesis $E(a_1) = E(A_1)$ was rejected in only one case, for $b \times \ell = 15.2$ cm $\times$ 121.9 cm . The ratios $\dfrac{\text{computed } t_0}{\pm_{0.975}(df_{\pm_0})}$ and $\dfrac{\text{computed } t_1}{\pm_{0.975}(df_{t_1})}$ took values only slightly greater than one for these six cases, but were only slightly less than one for several others. A more detailed examination of values of regression line results from Table 4.2 is in order.

Values of standard error of estimate s , slope a_0 , intercept a_1 , and the ratio $\dfrac{a_0}{a_1}$ from the regressions of each of data subgroups 2-7 are plotted against track width b in Figure 4.4 and against track length ℓ in Figure 4.5. Corresponding values from data subgroups 8-13 and 14-22 are plotted against index of soil strength G and load W , respectively, in Figures 4.6 and 4.7. Values of s , a_0 , and a_1 in Figures 4.4 and 4.7 appear to change in random fashion as functions of b and W , respectively. In Figure 4.6, the values of a_0 , a_1 , and $\dfrac{a_0}{a_1}$, do not show strong tendencies to depart from a random pattern. Values of s also appear not to be related to the value of G if results for the regression of the data subgroup with $6 \leq G < 7$ are ignored. The overall pattern of s versus G in Figure 4.6 certainly does not suggest a tendency of s to decrease as G increases. The fact that s for $6 \leq G < 7$ is based on only nine data points also

RELATIONS OF s, a_0, a_1, AND $\dfrac{a_0}{a_1}$ TO TRACK WIDTH b

FOR REGRESSION EQUATION $\dfrac{\widehat{P_{20}}}{W} = a_0 + a_1 \log \dfrac{G(b\ell)^{3/2}}{W}$

FOR DATA SUBGROUPS 2-7
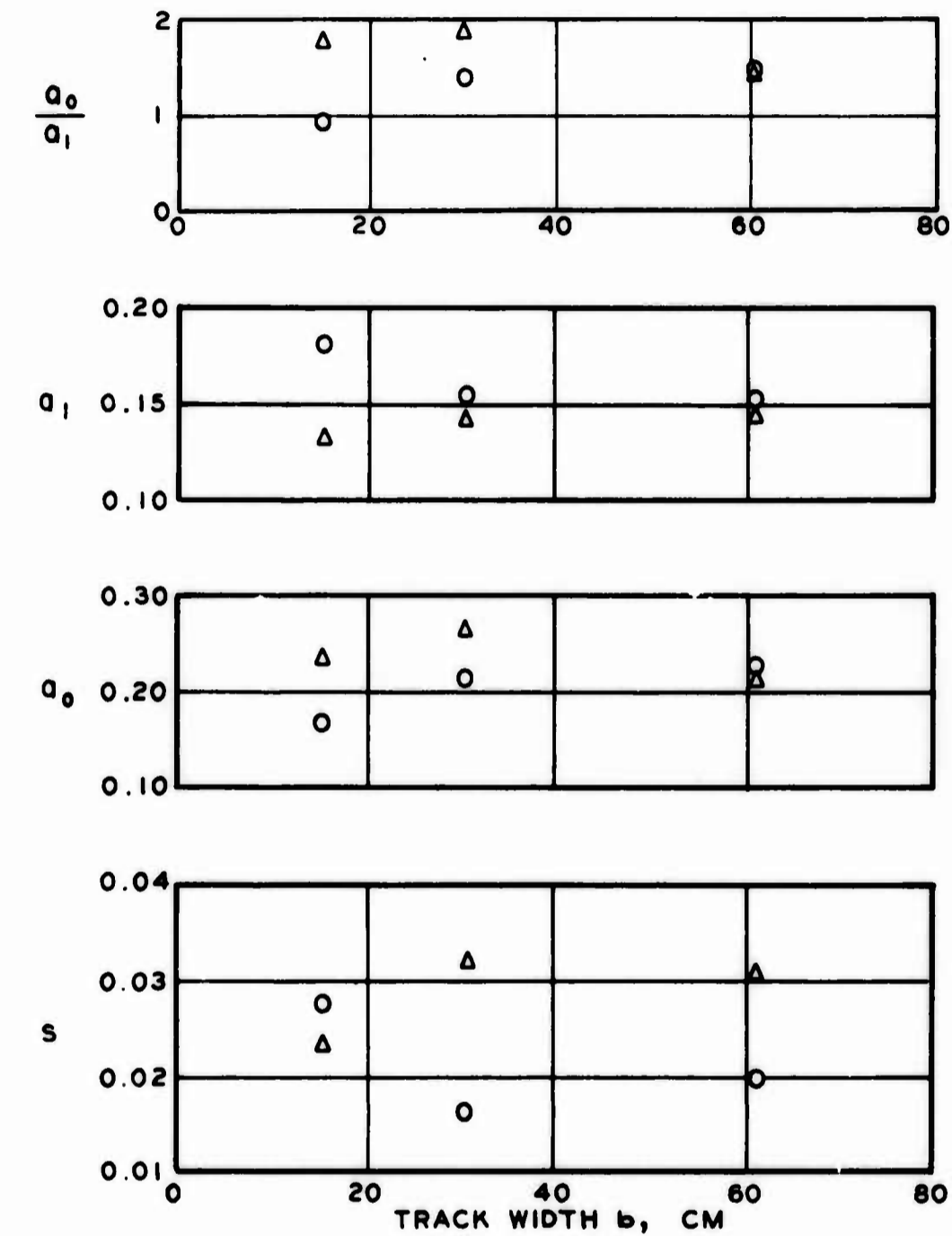
TRACK LENGTH, ℓ

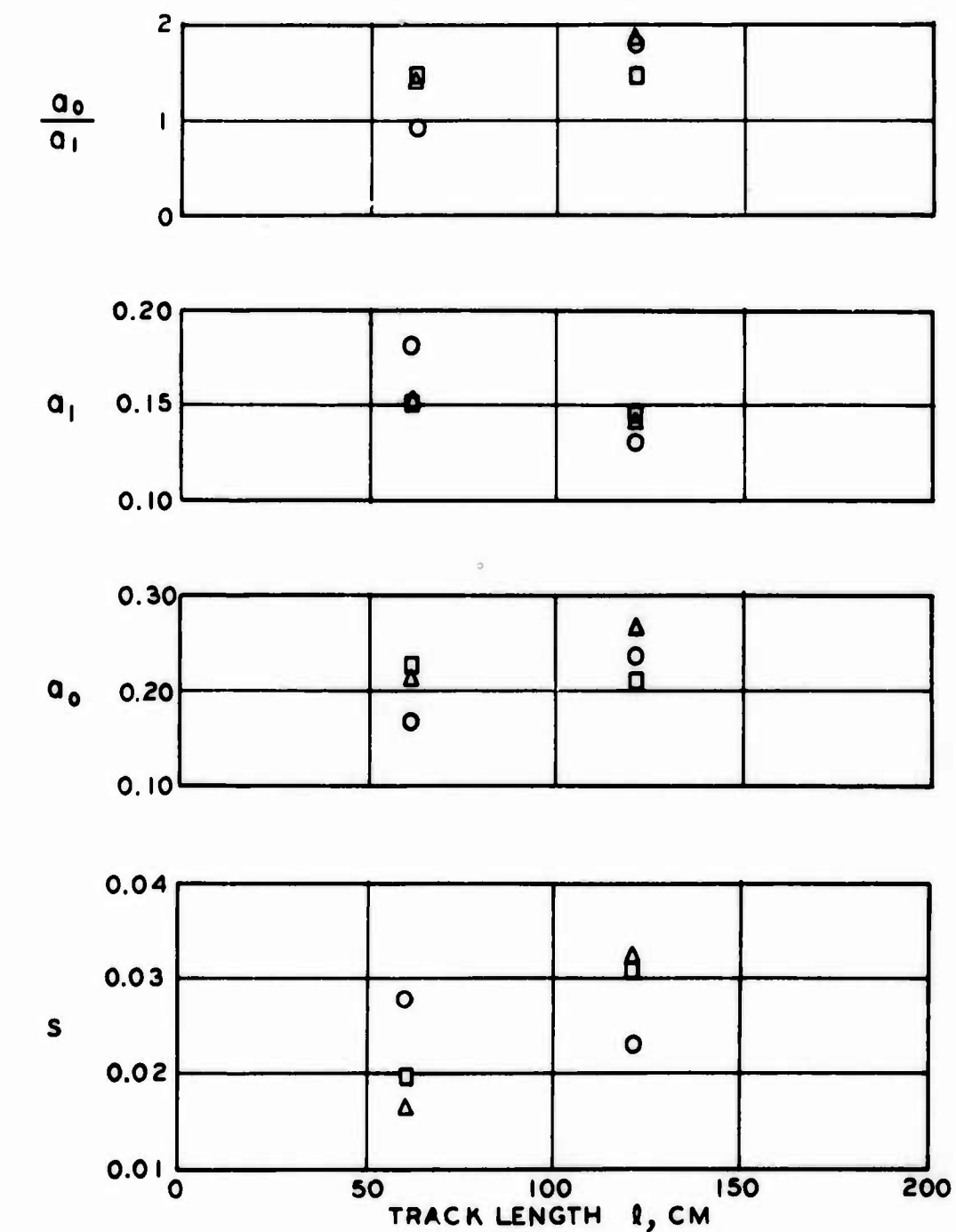| O | 61.0 CM |
| Δ | 121.9 CM |

NOTE: s = STANDARD ERROR OF ESTIMATE

a_0 = VALUE OF $\dfrac{P_{20}}{W}$ AT $\log \dfrac{G(b\ell)^{3/2}}{W} = 1.0$

a_1 = SLOPE

FIGURE 4.4

FIGURE 4.5

RELATIONS OF S, a_0, a_1, AND $\dfrac{a_0}{a_1}$ TO G FOR REGRESSION EQUATION $\dfrac{\widehat{P_{20}}}{W} = a_0 + a_1 \text{LOG} \dfrac{G(b\ell)^{3/2}}{W}$ FOR DATA SUBGROUPS 8-13

INDEX OF SAND STRENGTH G, $\dfrac{MN}{M^3}$

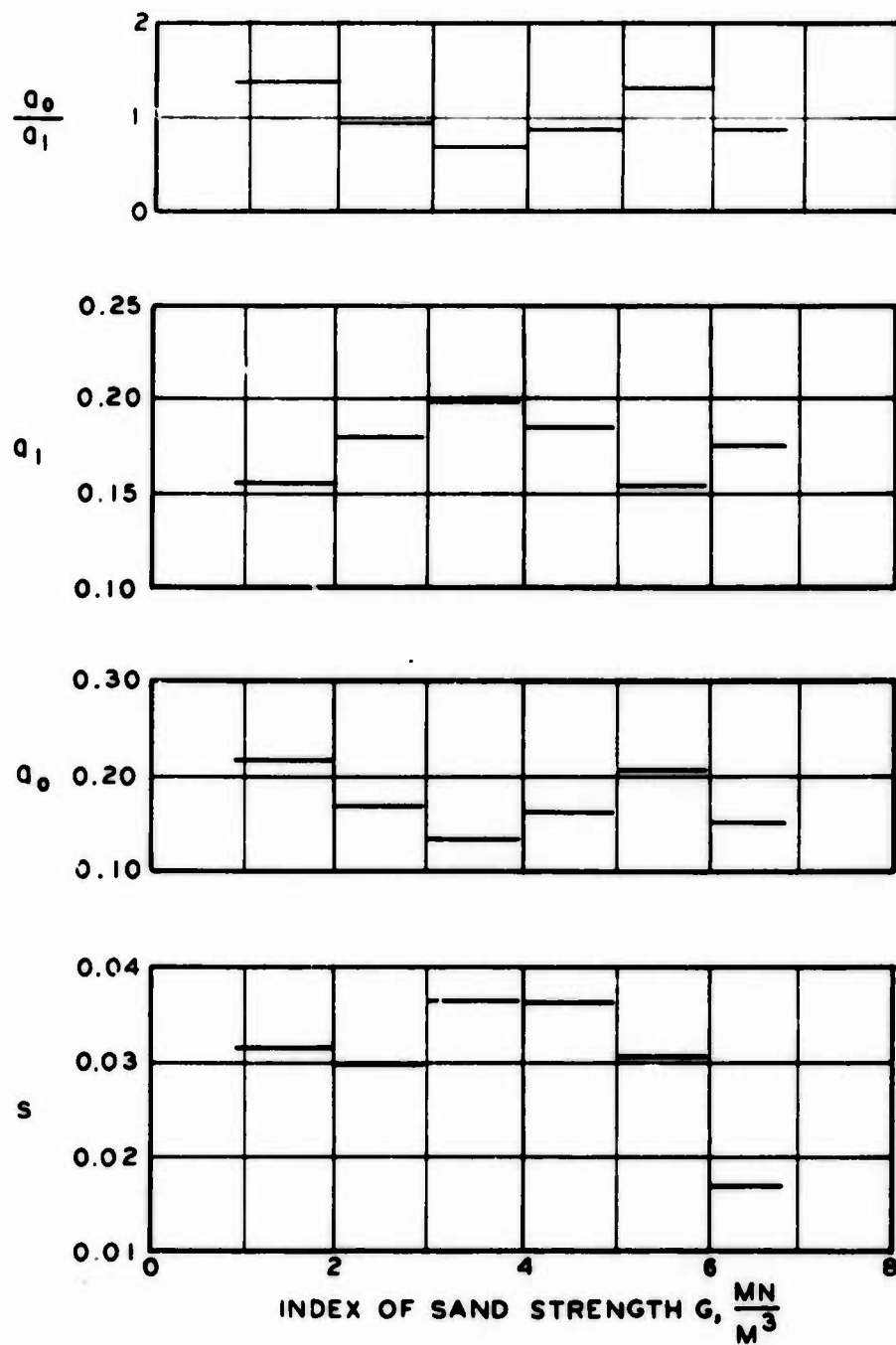NOTE: S = STANDARD ERROR OF ESTIMATE
a_0 = VALUE OF $\dfrac{P_{20}}{W}$ AT LOG $\dfrac{G(b\ell)^{3/2}}{W}$ = 1.0
a_1 = SLOPE

FIGURE 4.6

RELATIONS OF $S, a_0, a_1,$ AND $\frac{a_0}{a_1}$ TO LOAD W

FOR REGRESSION EQUATION $\frac{\hat{P_{20}}}{W} = a_0 + a_1 \text{LOG} \frac{G(b\ell)^{3/2}}{W}$

FOR DATA SUBGROUPS 14-22

NOTE: S = STANDARD ERROR OF ESTIMATE

$a_0 =$ VALUE OF $\frac{P_{20}}{W}$ AT $\text{LOG} \frac{G(b\,\ell)^{3/2}}{W} = 1.0$
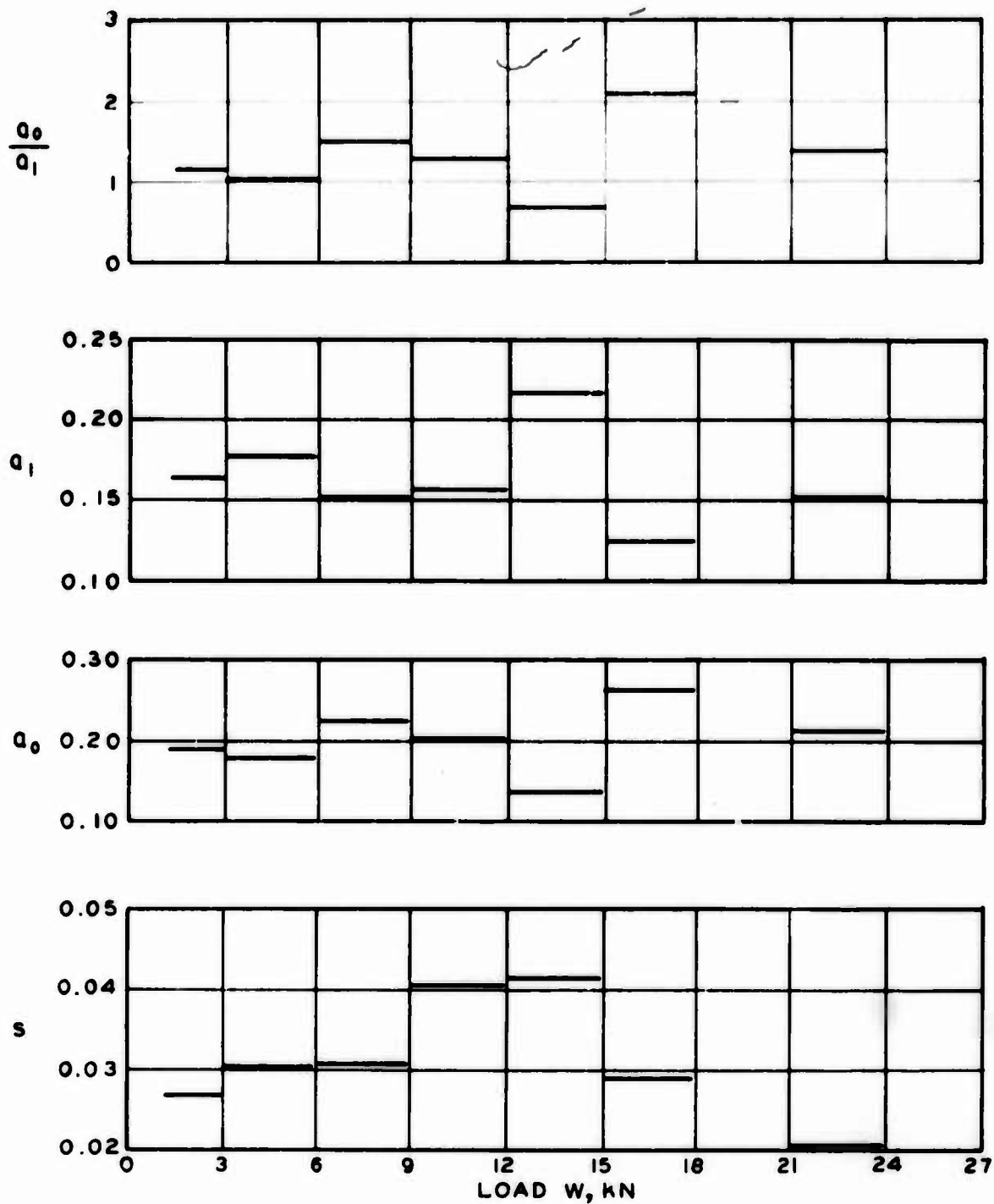
$a_1 =$ SLOPE

FIGURE 4.7

tends to support the contention that the small value of s for $6 \leq G < 7$ was the result of chance.

Only in Figure 4.5, where the relations of s , a_0 , a_1 , and $\frac{a_0}{a_1}$ to track length ℓ are presented, do there emerge distinguishable patterns. (Data plots and least-squares central lines on which the data of Figure 4.5 are based are presented in Figure 4.8.) As track length increased, the values of s generally increased. This tendency is opposite to what might be expected solely from a consideration of the physical sizes of tracks represented; i.e. as track length increases, the track might be expected to become generally more stable insofar as its ability to maintain a consistent level of performance is concerned. If the form of $\frac{G(b\ell)^{3/2}}{W}$ truly reflects the influence of ℓ on $\frac{P_{20}}{W}$, random patterns of s versus ℓ should emerge. None of the values of s are so large as to cause concern that the linear relations between $\frac{P_{20}}{W}$ and $\log \frac{G(b\ell)3/2}{W}$ for data subgroups 2-7 are not reasonably well defined; however, the pattern of s versus ℓ is somewhat unexpected.

In the upper three plots of Figure 4.5, values of a_0 and $\frac{a_0}{a_1}$ generally increase and those of a_1 decrease as the value of ℓ increases. (The significance of ratio $\frac{a_0}{a_1}$ will be discussed in Section C of Chapter IV.) In all plots of Figure 4.5, the relations of the Y-axis variable to ℓ are obscured by the presence of three levels of track width b for each value of ℓ . (A similar situation exists in Figure 4.4.) To gain insight into how the patterns of the middle two plots of Figure 4.5 were influenced by the b and ℓ main effects and the bℓ interaction, values of a_0 and a_1 were each treated
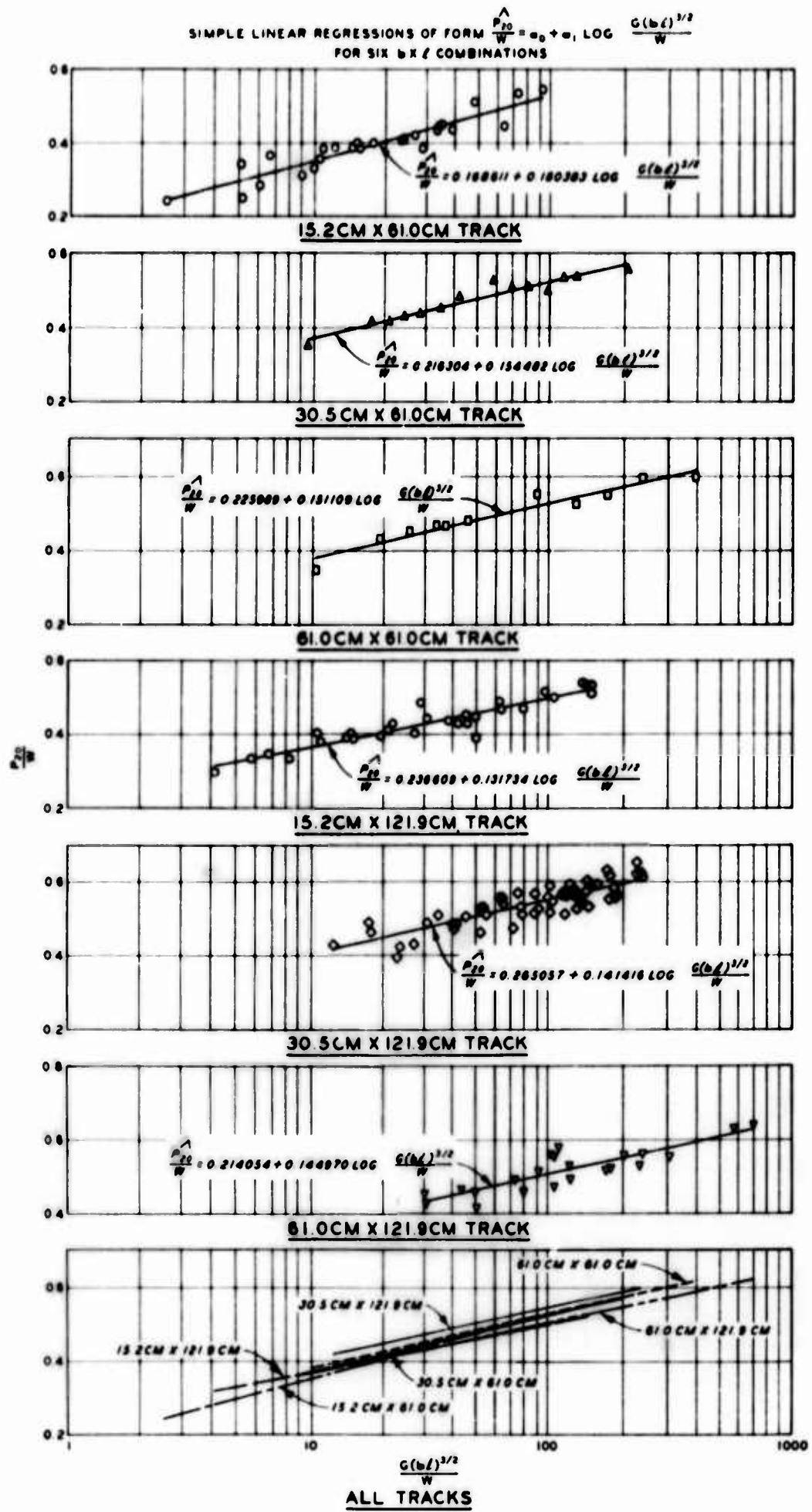
FIGURE 4.8

as results from a two-factor factorial experiment. Table 4.5 presents
this arrangement for a_0 . In this design, p = the number of levels

TABLE 4.5

TREATING a_0's AS RESPONSES IN A TWO-FACTOR FACTORIAL EXPERIMENT

	ℓ_1 = 61.0 cm	ℓ_2 = 121.9 cm	
b_1 = 15.2 cm	0.168611	0.236609	B_1 = 0.405220
b_2 = 30.5 cm	0.216304	0.265057	B_2 = 0.481811
b_3 = 61.0 cm	0.225989	0.214054	B_3 = 0.440043
	L_1 = 0.610904	L_2 = 0.716170	G = 1.327074

of b = 3 ; q = the number of levels of ℓ = 2 ; and n = the num-
ber of observations per entry , taken as one. For i rows and j
columns, values involved in working toward an analysis of variance are:

Correction Factor (CF) for the mean = $\dfrac{G^2}{npq} = \dfrac{(1.327074)^2}{1 \times 3 \times 2} = 0.293521$

$$SS_B = \sum_{i=1}^{3} \frac{B_i^2}{nq} - CF = \frac{(0.405220)^2 + (0.481811)^2 + (0.440043)^2}{1 \times 2} - 0.293521$$

$$= 0.001471$$

$$SS_L = \sum_{j=1}^{2} \frac{L_j^2}{np} - CF = \frac{(0.610904)^2 + (0.716170)^2}{1 \times 3} - 0.293521 = 0.001847$$

$$SS_{BL} = \sum_{i=1}^{3} \sum_{j=1}^{2} \frac{(BL_{ij})^2}{n} - (SS_B + SS_L + CF) = 0.001507$$

The value of $MS_{residuals}$, i.e. the variance associated with the a_0 values taken as entries in Table 4.5, can be estimated by pooling the values of $V(a_0)$ for data subgroups 2 through 7. From Table 4.3, the values of $V(a_0)$ for data subgroups 2 through 7 range from 0.0002245 through 0.0014970, so that an assumption of unequal variances of a_0's among these data subgroups is indicated. Since ANOVA is robust to departures from constant variance (Box [1954]), an assumption of equal variances should not influence the analysis too greatly. A pooled estimate for the $V(a_0)$'s then is given by $\dfrac{\sum\limits_{k}[(n-1)_k\ V(a_0)_k]}{\sum\limits_{k}(n-1)_k}$, where k designates subgroup number. Using data for subgroups 2-7 in Table 4.3 yields a pooled estimate of 0.000621. The analysis of variance is tabulated in Table 4.6. Values of $\dfrac{MS_B}{MS_{res}}$ and $\dfrac{MS_{BL}}{MS_{res}}$ are 1.2

TABLE 4.6

ANOVA FOR a_0's TREATED AS RESPONSES IN A

TWO-FACTOR FACTORIAL EXPERIMENT

Source	SS	df	MS
Total	Unknown	158	
CF	0.293521	1	
About $\overline{G}$	--	157	
B main effect	0.001471	2	0.000735
L main effect	0.001847	1	0.001847
BL interaction	0.001507	2	0.000754
Residuals	--	152	0.000621

and 1.2, respectively, and $F_{0.95(2,152)} = 3.0$. Thus, the hypotheses that b main effect and bl interaction do not affect a_0 are not

rejected at the 5 percent significance level. The calculated F-value for the hypothesis $H_0 = \sigma_\ell^2 = 0$ is $\frac{0.001847}{0.000621} = 3.0$. Because $F_{0.95(1,152)}$ = 3.9 , the hypothesis that ℓ does not affect a_0 is not rejected at the 5 percent significance level. However, the close proximity of the value of $\frac{MS_\ell}{MS_{res}}$ to that of $F_{0.95(1,152)}$ does cause some concern regarding the influence of the ℓ main effect on values of a_0 .

The analysis of the influence of b , ℓ , and $b\ell$ on a_1 follows a procedure parallel to that given above for a_0 . Values of a_1 arranged as responses in a two-factor factorial design are listed in Table 4.7 as follows:

TABLE 4.7

TREATING a_1's AS RESPONSES IN A TWO-FACTOR

FACTORIAL EXPERIMENT

	$\ell_1 = 61.0$ cm	$\ell_2 = 121.9$ cm	
$b_1 = 15.2$ cm	0.180383	0.131734	$B_1 = 0.312117$
$b_2 = 30.5$ cm	0.154482	0.141416	$B_2 = 0.295898$
$b_3 = 61.0$ cm	0.151109	0.144970	$B_3 = 0.296079$
	$L_1 = 0.485974$	$L_2 = 0.418120$	$G = 0.904094$

For i rows and j columns:

$$CF = \frac{G^2}{npq} = 0.136231$$

$$SS_B = \sum_{i=1}^{3} \frac{B_i^2}{nq} - CF = 0.000087$$

$$SS_L = \sum_{j=1}^{2} \frac{L_j^2}{np} - CF = 0.000767$$

$$SS_{BL} = \sum_{i=1}^{3} \sum_{j=1}^{2} \frac{(BL_{ij})^2}{n} - (SS_B + SS_L + CF) = 0.000520$$

$MS_{residuals}$ is estimated by assuming that the $V(a_1)$'s for data sub-groups 2-7 in Table 4.4 are equal, and pooling their values to obtain 0.000183. The results of ANOVA, then, are shown in Table 4.8. For the hypothesis $H_0 = \sigma_\ell^2 = 0$, $F_{calculated}$ is

TABLE 4.8

ANOVA FOR a_1's TREATED AS RESPONSES IN A TWO-FACTOR

FACTORIAL EXPERIMENT

Source	SS	df	MS
Total	Unknown	158	
CF	0.136231	1	
About G	--	157	
B main effect	0.000087	2	0.000044
L main effect	0.000767	1	0.000767
BL interaction	0.000520	2	0.000260
Residuals	--	152	0.000183

$\frac{0.000767}{0.000182} = 4.2$ and $F_{0.95(1,152)} = 3.9$. Thus, at the 5 percent

significance level, the test data cause rejection of the hypothesis that ℓ main effect does not influence the value of a_1 . Ratios $\frac{MS_B}{MS_{res}}$ and $\frac{MS_{BL}}{MS_{res}}$ take values 0.2 and 1.4, respectively, each considerably smaller than $F_{0.95(2,152)} = 3.0$.

Before contemplating an action based on the analyses of the effects of b , ℓ , and $b\ell$ on a_0 and a_1 , it is prudent to examine in more detail the arrangement of values of a_0 and a_1 in Tables 4.5 and 4.7, respectively. In Table 4.5, by far the smallest value of a_0 was obtained for $b = 15.2$ cm, $\ell = 61.0$ cm. The value for $b = 15.2$ cm, $\ell = 121.9$ cm was second largest of those obtained. In Table 4.7 the largest value of a_1 was obtained for $b = 15.2$ cm, $\ell = 61.0$ cm, and the smallest for $b = 15.2$ cm, $\ell = 121.9$ cm. Thus, the effects of b , ℓ , and $b\ell$ on values of a_0 and of a_1 would be reduced considerably if data for $b = 15.2$ cm were omitted. Values of a_0 and a_1 taken as responses in two-factor factorial designs with data for $b = 15.2$ cm omitted are presented in Table 4.9.

TABLE 4.9

TREATING a_0's AND a_1's AS RESPONSES IN TWO-FACTOR FACTORIAL EXPERIMENTS WITH DATA FOR $b = 15.2$ cm OMITTED

	For a_0		
	$\ell_1 = 61.0$ cm	$\ell_2 = 121.9$ cm	
$b_2 = 30.5$ cm	0.216304	0.265057	$B_1 = 0.481361$
$b_3 = 61.0$ cm	0.225989	0.214054	$B_2 = 0.440043$
	$L_1 = 0.442293$	$L_2 = 0.479111$	$G = 0.921404$

TABLE 4.9 (Concluded)

	$\ell_1 = 61.0$ cm	$\ell_2 = 121.9$ cm	
		For a_1	
$b_2 = 30.5$ cm	0.154482	0.141416	$B_1 = 0.295898$
$b_3 = 61.0$ cm	0.151109	0.144970	$B_2 = 0.296079$
	$L_1 = 0.305591$	$L_2 = 0.286386$	$G = 0.591977$

Pooled estimates of $V(a_0)$ and $V(a_1)$ were obtained by the same procedure as before, and the resulting analysis of variance is shown in Table 4.10. Because all ratios $\dfrac{MS_{calc}}{MS_{res}}$ in Table 4.10

TABLE 4.10

ANOVA'S FOR a_0's AND a_1's TREATED AS RESPONSES IN A

TWO-FACTOR FACTORIAL EXPERIMENT WITH DATA FOR

b = 15.2 cm OMITTED

Source	SS	df	MS	$\dfrac{MS_{calc}}{MS_{res}}$
		For a_0		
Total	Unknown	102		
CF	0.212246	1		
About $\bar{G}$	--	101		
B main effect	0.000427	1	0.000427	0.5
L main effect	0.000483	1	0.000483	0.6
BL interaction	0.000776	1	0.000776	1.0
Residuals	--	98	0.000790	

TABLE 4.10 (Concluded)

Source	SS	df	MS	$\frac{MS_{calc}}{MS_{res}}$
		For a_1		
Total	Unkrown	102		
CF	0.0876093	1		
About $\overline{G}$	--	101		
B main effect	0.0000003	1	0.0000003	0.0
L main effect	0.0000921	1	0.0000921	0.5
BL interaction	0.0000117	1	0.0000117	0.1
Residuals	--	98	0.0002011	

take values much smaller than $F_{0.95(1,98)} = 3.9$, the test data do not cause rejection of the hypotheses that effects b , l , and bl have no influence on a_0 and a_1 , at the 5 percent significance level, with . data for b = 15.2 cm ignored.

No definite answer is available to the question of why the effects of l on a_0 and l on a_1 are considerably lessened when data for the 15.2-cm-wide track are disregarded. One possibility is that the narrower the track, the more track pull is influenced by track-sand interactions along the edge of the track, as opposed to interactions beneath the track. Thus, 15.2 cm might be smaller than the lower limit of width that allows scaling of model to prototype. This argument cannot be totally discounted, but it seems weak on the basis of parallel work that WES has done for tires in sand. Using essentially the same approach as followed in Chapter III of this thesis, Freitag [1965] developed a dimensionless prediction term that can be used to predict with good accuracy the in-sand $\frac{P_{20}}{W}$ performance of tires whose hard-surface con-tact widths range from 5 to 40 cm and more.

The analysis of data in Tables 4.5 through 4.10 provided insight into the causes for differences between values of various parameters of the linear regression $\dfrac{\hat{P}_{20}}{W} = a_0 + a_1 \log \dfrac{G(b\ell)^{3/2}}{W}$. The overall implication was that whatever changes in the form of $\dfrac{G(b\ell)^{3/2}}{W}$ that might lead to improvement in the term's ability to predict $\dfrac{P_{20}}{W}$ would involve b and ℓ . Accordingly, several dimensionless forms of the basic-variable prediction term were developed by holding values of the exponents of G and W at $+1$ and -1 , respectively (no consistent pattern of effects of either G or W on either a_0 or a_1 were indicated in Figures 4.6 and 4.7) and varying the values of exponents of b and ℓ , all the while retaining the dimensionless character of the overall prediction term. Table 4.11 presents results obtained from linear regressions of form $\dfrac{\hat{P}_{20}}{W} = a_0 + a_1 \log \dfrac{Gb^{x_1}\ell^{x_2}}{W}$.

TABLE 4.11

COMPARISON OF VALUES FROM SIMPLE LINEAR REGRESSIONS OF FORM $\dfrac{\hat{P}_{20}}{W} = a_0 + a_1 \log \dfrac{Gb^{x_1}\ell^{x_2}}{W}$

$\dfrac{\hat{P}_{20}}{W} = a_0 + a_1 \log \dfrac{Gb^{x_1}\ell^{x_2}}{W}$				Sum of Squares				
				About the Mean	About Regression	Due to Regression	Standard Error	Coefficient of
x_1	x_2	a_0	a_1	$\Sigma(Y_i - \bar{Y})^2$	$\Sigma(Y_i - Y_{est})^2$	$\Sigma(Y_{est} - \bar{Y})^2$	of Estimate s	Correlation r
0	3.00	0.208758	0.107590	1.087021	0.593041	0.493980	0.061657	0.674117
0.50	2.50	0.164791	0.139930	1.087021	0.394233	0.692788	0.050271	0.798328
1.00	2.00	0.159897	0.161602	1.087021	0.228824	0.858196	0.038299	0.888535
1.20	1.80	0.172479	0.164335	1.087021	0.190671	0.896350	0.034961	0.908071
1.40	1.60	0.192374	0.163294	1.087021	0.172862	0.914159	0.033288	0.917408
1.45	1.55	0.198277	0.162483	1.087021	0.171560	0.915461	0.033162	0.917701
1.50	1.50	0.204482	0.161407	1.087021	0.171459	0.915562	0.033153	0.917751
1.55	1.45	0.210955	0.160267	1.087021	0.172516	0.914505	0.033255	0.917221
1.60	1.40	0.217658	0.158887	1.087021	0.174680	0.912341	0.033463	0.916135
1.80	1.20	0.246061	0.151858	1.087021	0.193198	0.893823	0.035192	0.906790
2.00	1.00	0.275458	0.143078	1.087021	0.224295	0.862726	0.037918	0.890876
2.50	0.50	0.343668	0.118444	1.087021	0.330237	0.756784	0.046010	0.834386
3.00	0	0.396650	0.095705	1.087021	0.441110	0.645911	0.053175	0.770846

Values obtained for each of r , s , and SS about regression indicate the best arrangement of x_1 and x_2 in $\dfrac{Gb^{x_1}\ell^{x_2}}{W}$ as values of x_1 and

x_2 both approach 1.5. This result is illustrated in Figure 4.9 by the relation of s to x_1. Thus, further support is developed for the general conclusion from Table 4.10, i.e. a major part of variation in the relation of $\dfrac{P_{20}}{W}$ to $\log \dfrac{G(b\ell)^{3/2}}{W}$ is caused by data for the narrowest track. From this and the several other analyses in Section A of Chapter IV, no strong evidence was developed to indicate that substantial improvement in ability to predict $\dfrac{P_{20}}{W}$ could be made by changing the form of the basic-variable prediction term $\dfrac{G(b\ell)^{3/2}}{W}$.

B. **An alternate basic-variable prediction term**

Another means of diagnosing whether the test data suggest $\dfrac{P_{20}}{W}$ is predicted more accurately by a term composed of multiplicative
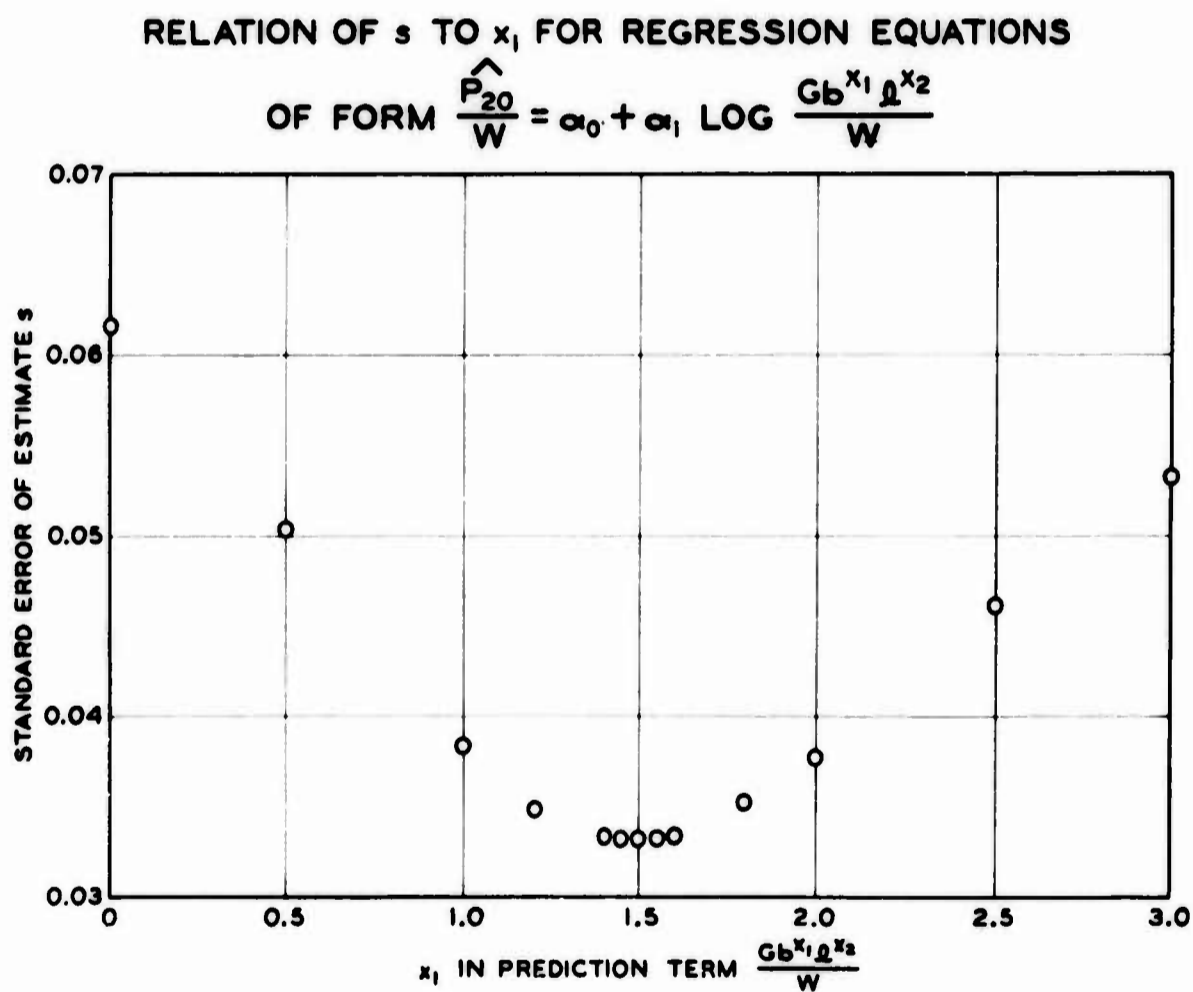

RELATION OF s TO x_1 FOR REGRESSION EQUATIONS OF FORM $\dfrac{\widehat{P_{20}}}{W} = \alpha_0 + \alpha_1 \log \dfrac{Gb^{x_1}\ell^{x_2}}{W}$

FIGURE 4.9

functions of G , b , ℓ , and W other than $\frac{G(b\ell)^{3/2}}{W}$ is to use a multiple linear regression to develop a prediction equation of a form such that the effectiveness of $\frac{G(b\ell)^{3/2}}{W}$ and the result from the multiple linear regression can be compared. The linear regression equation that relates $\frac{P_{20}}{W}$ to $\log \frac{G(b\ell)^{3/2}}{W}$ for all 158 data points is

$$\frac{\hat{P}_{20}}{W} = 0.204482 + 0.161470 \log \frac{G(b\ell)^{3/2}}{W} \ . \qquad [4.B.1]$$

This equation can be expressed as

$$\frac{\hat{P}_{20}}{W} = 0.204482 + 0.161470 \log G - 0.161470 \log W$$

$$+ \ 0.242205 \log b + 0.242205 \log \ell \ . \quad [4.B.2]$$

Equation [4.B.2] can be transformed to

$$\left(\frac{\hat{P}_{20}}{W} - 0.4828\right) = 0.161470 \, (\log G - 0.4271) - 0.161470 \, (\log W - 3.8468)$$

$$+ \ 0.242205 \, (\log b - 1.4400) + 0.242205 \, (\log \ell - 1.9890) \ , \qquad [4.B.3]$$

which is of the form

$$(\hat{Y} - \overline{Y}) = a_1(X_1 - \overline{X}_1) + a_2(X_2 - \overline{X}_2) + a_3(X_3 - \overline{X}_3) + a_4(X_4 - \overline{X}_4) \ . [4.B.4]$$

Equation [4.B.4] eliminates the intercept term a_o by transposing the test data such that the equation of the line passes through $(X_1 = 0$, $X_2 = 0$, $X_3 = 0$, $X_4 = 0$; $Y = 0)$.

Next, a multiple linear regression of dependent variable $\frac{P_{20}}{W}$ and independent variables $\log G$, $\log W$, $\log b$, and $\log \ell$ was used to develop an equation of a form comparable to [4.B.2].

The equation thus obtained is

$$\frac{\hat{P}_{20}}{W} = 0.177491 + 0.141601 \log G - 0.169701 \log W$$

$$+ 0.248757 \log b + 0.271217 \log \ell . \qquad [4.B.5]$$

Data pertinent to the multiple regression used to obtain equation [4.B.5] are summarized in Table 4.12. Equation [4.B.5] can be expressed in the form of [4.B.4] as

$$\left(\frac{\hat{P}_{20}}{W} - 0.4828\right) = 0.141601 (\log G - 0.4271) - 0.169701 (\log W - 3.8468)$$

$$+ 0.248757 (\log b - 1.4400) + 0.271217 (\log \ell - 1.9890) . \qquad [4.B.6]$$

The similarity of values of corresponding coefficients in equations [4.B.3] and [4.B.6] is obvious.

The residuals obtained from the multiple linear regression to obtain equation [4.B.5] are plotted on normal probability paper in Figure 4.10. A corresponding plot for equation [4.B.1] appears in Figure 4.3. The distribution of residuals in these two figures can be closely approximated by straight lines of almost identical slope passing through (0, 50), indicating very similar normal distributions centered on zero.

A more definitive means of testing whether equation [4.B.5] describes the relation between $\frac{P_{20}}{W}$ and $\log G$, $\log W$, $\log b$, and $\log \ell$ better than does equation [4.B.1] is to assess the improvement in this relation caused by estimating five coefficients in equation [4.B.5] versus two in equation [4.B.1]. The partial F-test criterion from Draper and Smith [1966] can be applied at the 5 percent

TABLE 4.12

MULTIPLE LINEAR REGRESSION OF $\dfrac{P_{20}}{W}$ ON

LOG G , LOG b , LOG ℓ , LOG W

Variable	Mean	Standard Deviation
Log b	1.43997	0.222934
Log G	0.42708	0.252037
Log W	3.84680	0.276024
Log ℓ	1.98895	0.141024
P_{20}/W	0.48280	0.083209

Correlation Matrix

	Log b	Log G	Log W	Log ℓ	$\dfrac{P_{20}}{W}$
Log b	1.000000	-0.084748	0.375970	0.139611	0.482651
Log G	-0.084748	1.000000	-0.212299	-0.048155	0.469801
Log W	0.375970	-0.212299	1.000000	0.283270	-0.273212
Log ℓ	0.139611	-0.048155	0.283270	1.000000	0.372595
P_{20}/W	0.482651	0.469801	-0.273212	0.372595	1.000000

Variance-Covariance Matrix

	Log b	Log G	Log W	Log ℓ	$\dfrac{P_{20}}{W}$
Log b	0.002263910	-0.000031590	-0.000129174	-0.000285055	-0.000532886
Log G	-0.000031590	0.000161152	0.000000800	-0.000047450	-0.000009189
Log W	-0.000129174	0.000000800	0.000113233	0.000022095	-0.000002682
Log ℓ	-0.000285055	-0.000047450	0.000022095	0.000116374	-0.000052148
P_{20}/W	-0.000532886	-0.000009189	-0.000002682	-0.000052148	0.000376011

$$\frac{\widehat{P_{20}}}{W} = a_0 + a_1 \, \text{Log } b + a_2 \, \text{Log } G + a_3 \, \text{Log } W + a_4 \, \text{Log } \ell$$

Term	Value	Standard Error	t-Ratio
a_0	0.177491	0.047581	3.73
a_1	0.248757	0.012695	19.60
a_2	0.141601	0.010641	13.31
a_3	-0.169701	0.010788	-15.73
a_4	0.271217	0.019391	13.99

Coefficient of Multiple Determination, $R^2 = 0.844279$
Coefficient of Multiple Correlation, $R = 0.918847$
Standard Error of Estimate, $s = 0.032835$
Number of Data Points, $n = 158$
Degrees of Freedom, $n - p - 1 = 153$
Residual Sum of Squares = 0.1650

NORMAL PROBABILITY PLOT OF RESIDUALS
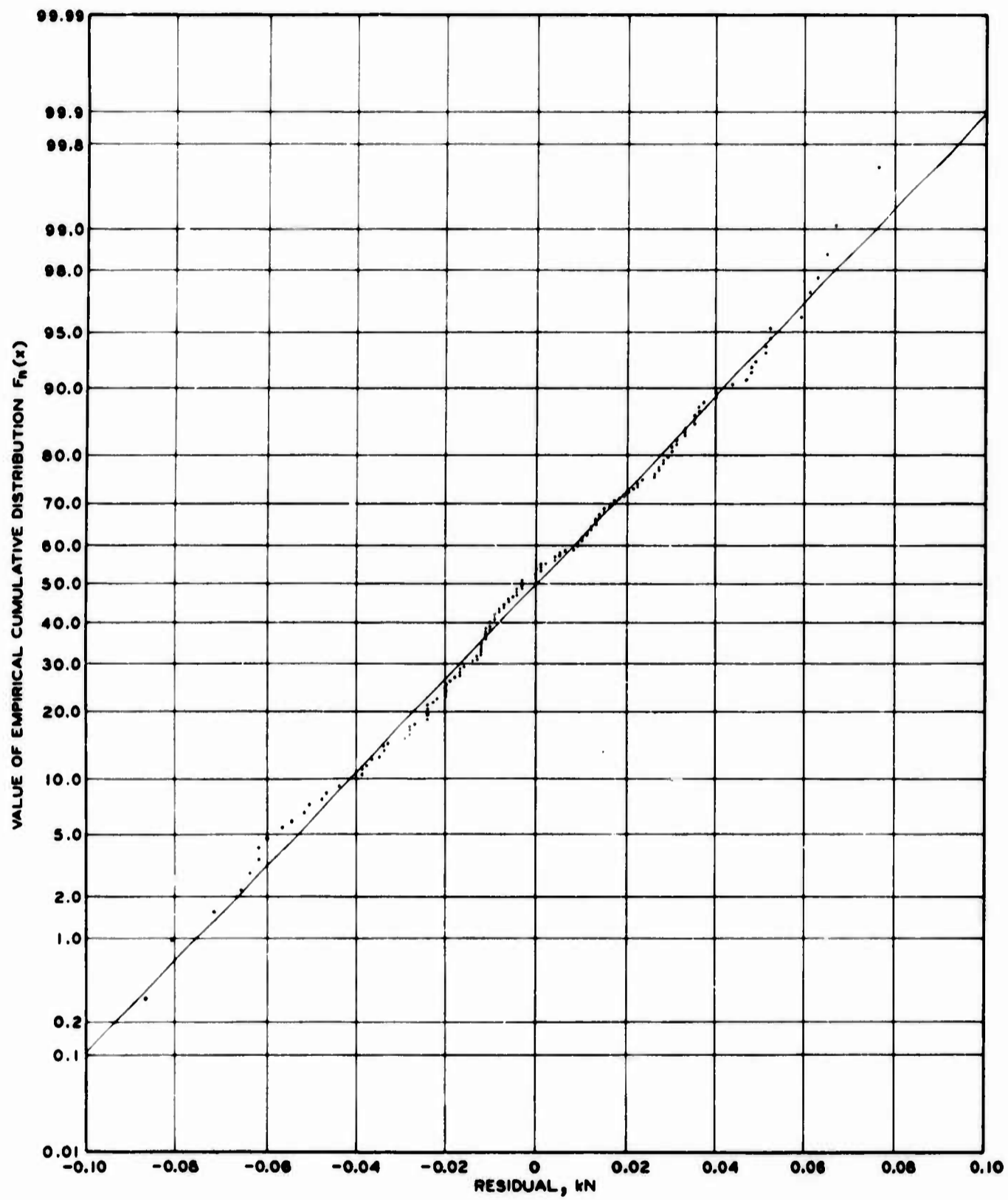FROM APPLICATION OF EQUATION [4.B.5]

FIGURE 4.10

significance level by comparing the quantity

$$\frac{(SS_{res} \text{ from eq } [4.B.1] - SS_{res} \text{ from eq } [4.B.5]) + 3df}{MS_{res} \text{ from eq } [4.B.5]}$$

against $F_{0.95(3,153)} = 2.7$. SS_{res} is estimated as $s^2(n - p - 1)$ and MS_{res} as s^2. Based on values from Tables 4.2 and 4.12, computed F is $\frac{[(0.001099(156) - 0.001078(153))] + 3}{0.001078} = 2.0$. Thus, the hypothesis that equations [4.B.1] and [4.B.5] produce equally good estimates of the relation between $\frac{P_{20}}{W}$ and $\log G$, $\log W$, $\log b$, and $\log \ell$ is not rejected at the 5 percent significance level.

In summary, comparisons of regression equations [4.B.1] and [4.B.5] and of coefficients that appear in equations [4.B.3] and [4.B.6] showed that: (a) values of each of the four coefficients of equation [4.B.3] closely match those of corresponding coefficients in equation [4.B.6]; (b) the distributions of residuals obtained from application of equations [4.B.1] and [4.B.5] are very similar; and (c) no significant difference was indicated in the ability of equations [4.B.1] and [4.B.5] to predict $\frac{P_{20}}{W}$. Thus, the test data do not appear to cause rejection of the hypothesis that regression equation [4.B.1] predicts $\frac{P_{20}}{W}$ as accurately as does equation [4.B.5]. On the basis of this conclusion, and because $\frac{G(b\ell)^{3/2}}{W}$ is indicated by consideration of similitude to be able to predict the $\frac{P_{20}}{W}$ performance of prototype tracks operating under conditions similar to those under which the model tracks were tested, $\frac{G(b\ell)^{3/2}}{W}$ is retained as the basic-variable prediction term for tracks operating in sand.

C. Relations among P_{20} , G , b , ℓ , and W
for the maximum pull and zero pull conditions

The dimensionless form of the terms $\dfrac{P_{20}}{W}$ and $\dfrac{G(b\ell)^{3/2}}{W}$ has the tendency to obscure the relations between dependent parameter P_{20} and independent parameters G , b , ℓ , and W . In particular, it is of interest to determine how values of P_{20} change relative to those of load W for a given combination of track size and index of soil strength G . The general form of the linear regression equation relating $\dfrac{P_{20}}{W}$ to $\log \dfrac{G(b\ell)^{3/2}}{W}$ is

$$\frac{\widehat{P_{20}}}{W} = a_0 + a_1 \log \frac{G(b\ell)^{3/2}}{W} \; . \qquad [4.\text{C}.1]$$

If the description of a particular track-sand situation is given by the value $G(b\ell)^{3/2}$, denoted hereafter by the letter k , then

$$\frac{\widehat{P_{20}}}{W} = a_0 + a_1 \log \frac{k}{W}$$

and

$$\widehat{P_{20}} = a_0 W + (a_1 \log k) \, W - (a_1 \log W) \, W \; . \qquad [4.\text{C}.2]$$

To determine the value of load W^* that produces maximum P_{20} (i.e., $P_{20}^{\,*}$), the derivative of $\widehat{P_{20}}$ with respect to W^* is set equal to 0 yielding

$$\frac{d\widehat{P_{20}^{\,*}}}{dW^*} = a_0 + a_1 \log k - \left[a_1 \log W^* + W^* \left(\frac{a_1 \log e}{W^*} \right) \right] = 0 \; . \qquad [4.\text{C}.3]$$

Then,

$$\hat{W}^* = 10^{a_0/a_1} \times \frac{k}{e} \ . \qquad\qquad [4.C.4]$$

If $10^{a_0/a_1} \times \frac{k}{e}$ is substituted for W in equation [4.C.2],

$$\hat{P}_{20}^* = \left(10^{a_0/a_1} \times \frac{k}{e}\right) \times \left[a_0 + a_1 \log k - a_1 \log \left(10^{a_0/a_1} \times \frac{k}{e}\right)\right]$$

$$= \left(10^{a_0/a_1} \times \frac{k}{e}\right) \times \left(a_1 \log e\right) \ . \qquad\qquad [4.C.5]$$

Taking the ratio of values of $\hat{P}_{20}^*$ and $\hat{W}^*$ yields

$$\frac{\hat{P}_{20}^*}{\hat{W}^*} = \frac{\left(10^{a_0/a_1} \times \frac{k}{e}\right) \times (a_1 \log e)}{10^{a_0/a_1} \times \frac{k}{e}} = a_1 \log e = 0.4343 a_1 \qquad [4.C.6]$$

For $W^* = 10^{a_0/a_1} \times \frac{k}{e}$, $\dfrac{d\hat{P}_{20}^*}{dW^*}$ vanishes and $\dfrac{d^2\hat{P}_{20}^*}{dW^{*2}} = -\dfrac{a_1 e \log e}{10^{a_0/a_1} \times k}$;

thus, the second derivative test verifies a maximum value of P_{20} at $\hat{W}^* = 10^{a_0/a_1} \times \frac{k}{e}$. To illustrate the relation of $\hat{P}_{20}^*$ to $\hat{W}^*$, consider an example where $b = 15.2$ cm, $\ell = 121.9$ cm, $G = 1.0$ MN/m^3, and $\dfrac{\hat{P}_{20}}{W} = 0.204482$ to $0.161470 \log \dfrac{G(b\ell)^{3/2}}{W}$. Figure 4.11 shows that the relation of $\hat{P}_{20}^*$ to $\hat{W}^*$ is roughly parabolic in shape, and that for the conditions of this example, $\hat{P}_{20}^*$ is predicted to have a maximum value at $\dfrac{\hat{P}_{20}^*}{\hat{W}^*} = 0.4343 a_1 = 0.07013$.

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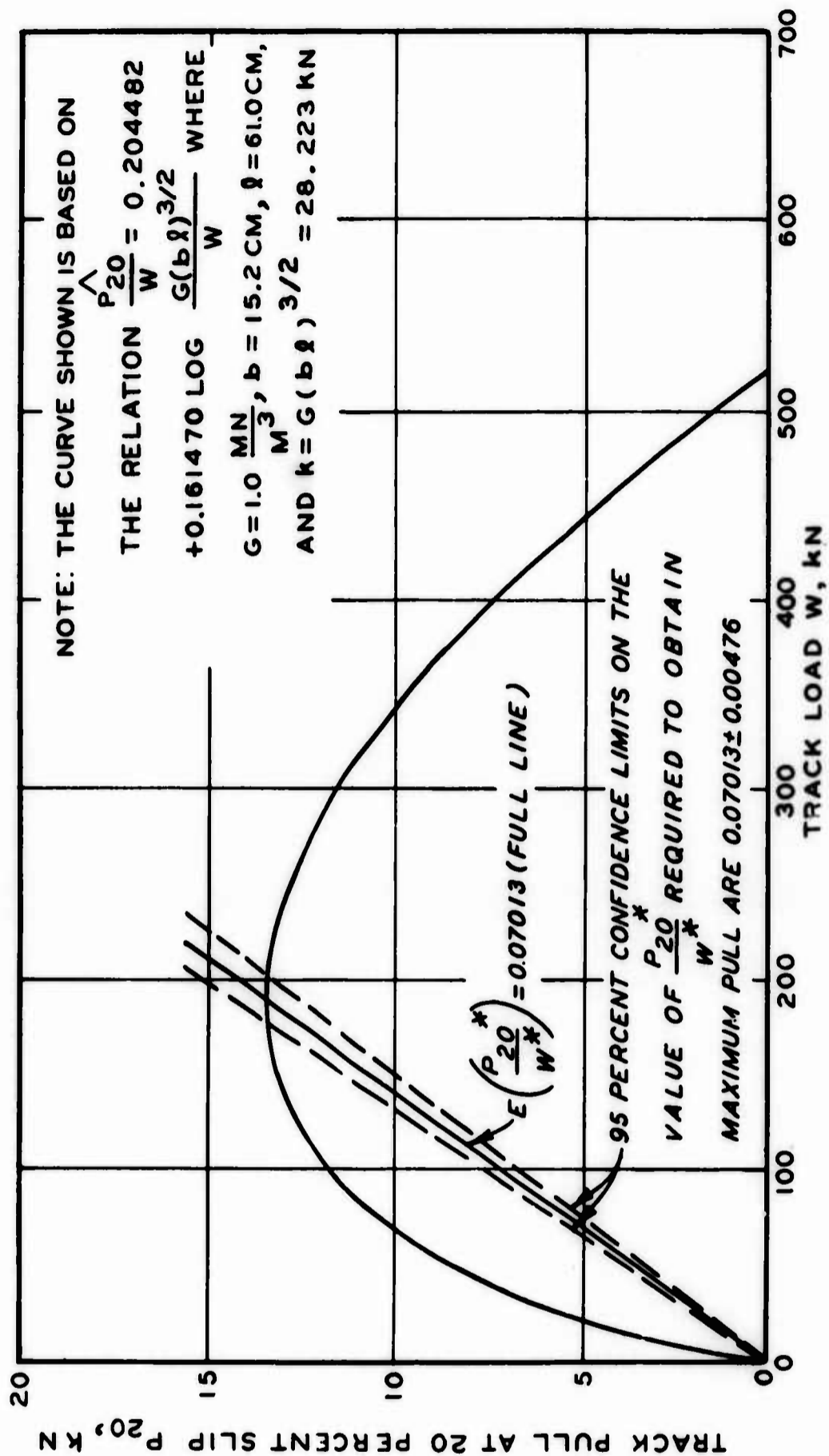RELATION OF $\hat{P}_{20}^*$ TO $\hat{W}^*$ FOR A PARTICULAR VALUE OF $G(b\ell)^{3/2}$

FIGURE 4.11

To obtain an estimate of the precision of the value of $\dfrac{\hat{P}_{20}^{*}}{\hat{W}^{*}}$
that produces maximum P_{20} , recall from equation [4.C.6] that the

$\dfrac{\hat{P}_{20}^{*}}{\hat{W}^{*}}$ that corresponds to maximum P_{20} is defined by $0.4343a_{1}$.

Davies [1961] defines $100(1 - 2\alpha)$ percent confidence limits for slope
$E(a_{1})$ of a simple linear regression analysis as $a_{1} \pm t_{\alpha} \dfrac{s}{\sqrt{\Sigma(X_{i} - \bar{X})^{2}}}$.

From Table 4.2 for data subgroup 1 (158 data points), $a_{1} = 0.161470$,

$s = 0.033153$, and $s_{x} = \sqrt{\dfrac{\Sigma(X_{i} - \bar{X})^{2}}{N - 1}} = 0.472936$. Then, $\dfrac{s}{\sqrt{\Sigma(X_{i} - \bar{X})^{2}}}$

$= \dfrac{0.033153}{\sqrt{157} \times (0.472936)} = 0.0055945$, and 95 percent confidence limits on

the value of $\dfrac{\hat{P}_{20}^{*}}{\hat{W}^{*}}$ that produces maximum P_{20} are $0.4343(0.161470)$

$\pm 1.96 \, (0.4343 \times 0.0055945)$, or 0.07013 ± 0.00476 .

It is important, also, to estimate the value of load W_{I} that
will cause the track to become immobilized, i.e. the W_{I} that will
produce $P_{20} = 0$. From equation [4.C.2],

$$\frac{\hat{P}_{20}}{W_{I}} = a_{0} + a_{1} \log k - a_{1} \log W_{I} = 0 \ ,$$

or

$$\log \hat{W}_{I} = \frac{a_{0}}{a_{1}} + \log k \ ,$$

and

$$\widehat{W}_I = 10^{a_0/a_1} \times k \ . \qquad\qquad [4.\text{C}.7]$$

In Figure 4.11, $\widehat{W}_I = 10^{1.26638} \times 28.233$ kN = 521 kN .

Confidence limits on $E(W_I)$ are more readily obtained by dealing with the equation for $\log \widehat{W}_I$ than for $\widehat{W}_I$. [Note from equation [4.C.4] that $\log (\widehat{W}^* \text{ for maximum } P_{20})$ equals $\dfrac{a_0}{a_1} + \log \dfrac{k}{e}$; thus, $\log (\widehat{W} \text{ for maximum } P_{20})$ and $\log (\widehat{W} \text{ for } P_{20} = 0)$ differ only in the denominator of the value added to $\dfrac{a_0}{a_1}$.] From Davies [1961], Fieller's Theorem defines confidence limits for the true value of a ratio $\dfrac{a_0}{a_1}$, where a_0 and a_1 are unbiased estimates, each assumed normally distributed, as follows:

$$\left[\frac{a_0}{a_1} - \frac{t^2 \text{ cov } (a_0, a_1)}{a_1^2} \pm \frac{t}{a_1} \right.$$

$$\times \left. \sqrt{V(a_0) - \frac{2a_0}{a_1} \text{ cov } (a_0, a_1) + \frac{a_0^2}{a_1^2} V(a_1) - \frac{t^2 V(a_1)}{a_1^2} \left\{ V(a_0) - \frac{[\text{cov}(a_0, a_1)]^2}{V(a_0)} \right\}} \right]$$

$$+ 1 - \frac{t^2 V(a_1)}{a_1^2} \ . \qquad [4.\text{C}.8]$$

Values of $V(a_0)$, $V(a_1)$, and $\text{cov}(a_0, a_1)$ are defined from Draper and Smith [1966] as

$$V(a_0) = \frac{\sigma^2 \Sigma X_i^2}{n\Sigma(X_i - \bar{X})^2} \ , \quad V(a_1) = \frac{\sigma^2}{\Sigma(X_i - \bar{X})^2} \ , \quad \text{and} \quad \text{cov}(a_0, a_1) = \frac{-\sigma^2 \bar{X}}{\Sigma(X_i - \bar{X})^2} \ .$$

The following values are taken from Table 4.2 for data subgroup 1 (all 158 data points):

$$s^2 = \text{an estimate of } \sigma^2 = (0.033153)^2 = 0.00109912$$

$$\Sigma(X_i - \bar{X})^2 = s_X^2 \times (n-1) = (0.472936)^2 \times 157 = 35.5869$$

$$\Sigma X_i = n\bar{X} = 158(1.72367) = 272.340$$

$$\Sigma X_i^2 = 504.541 \left(\text{all with } X = \log \frac{G(b\ell)^{3/2}}{W}\right).$$

Substituting these values in the expressions for $V(a_0)$, $V(a_1)$, and $cov(a_0,a_1)$ produces the following estimates: $V(a_0) = 0.000008627$; $V(a_1) = 0.000030886$; and $cov(a_0,a_1) = -0.000053236$. With intercept $a_0 = 0.204482$, slope $a_1 = 0.161470$, $t = t_{(\alpha/2,n-2)} = t_{(0.025,156)} = 1.960$, and the values given above for $V(a_0)$, $V(a_1)$ and $cov(a_0,a_1)$, 95 percent confidence limits for $E\left(\dfrac{a_0}{a_1}\right)$ are computed from equation [4.C.8] to be

$$\frac{1.27422 \pm 0.20408}{0.9963412} = 1.27890 \pm 0.20483 = 1.07407 \text{ to } 1.48373 .$$

In the equation $\log \widehat{W}_I = \dfrac{a_0}{a_1} + \log k$, the term k equals $G(b\ell)^{3/2}$. Values of track length and width, b and ℓ , respectively, are measured under static conditions cn a flat, rigid surface so that practically no variation is associated with their measured (estimated) values. Values of index of sand strength G do vary slightly over the length of a test bed. Standard deviations of $\log G$ measurements were found to be relatively constant for the 158 test beds considered (see Figure 4.12), and a pooled estimate of form $\dfrac{\sum\limits_{q}(n-1)s^2}{\sum\limits_{q}(n-1)}$ for $q = 158$
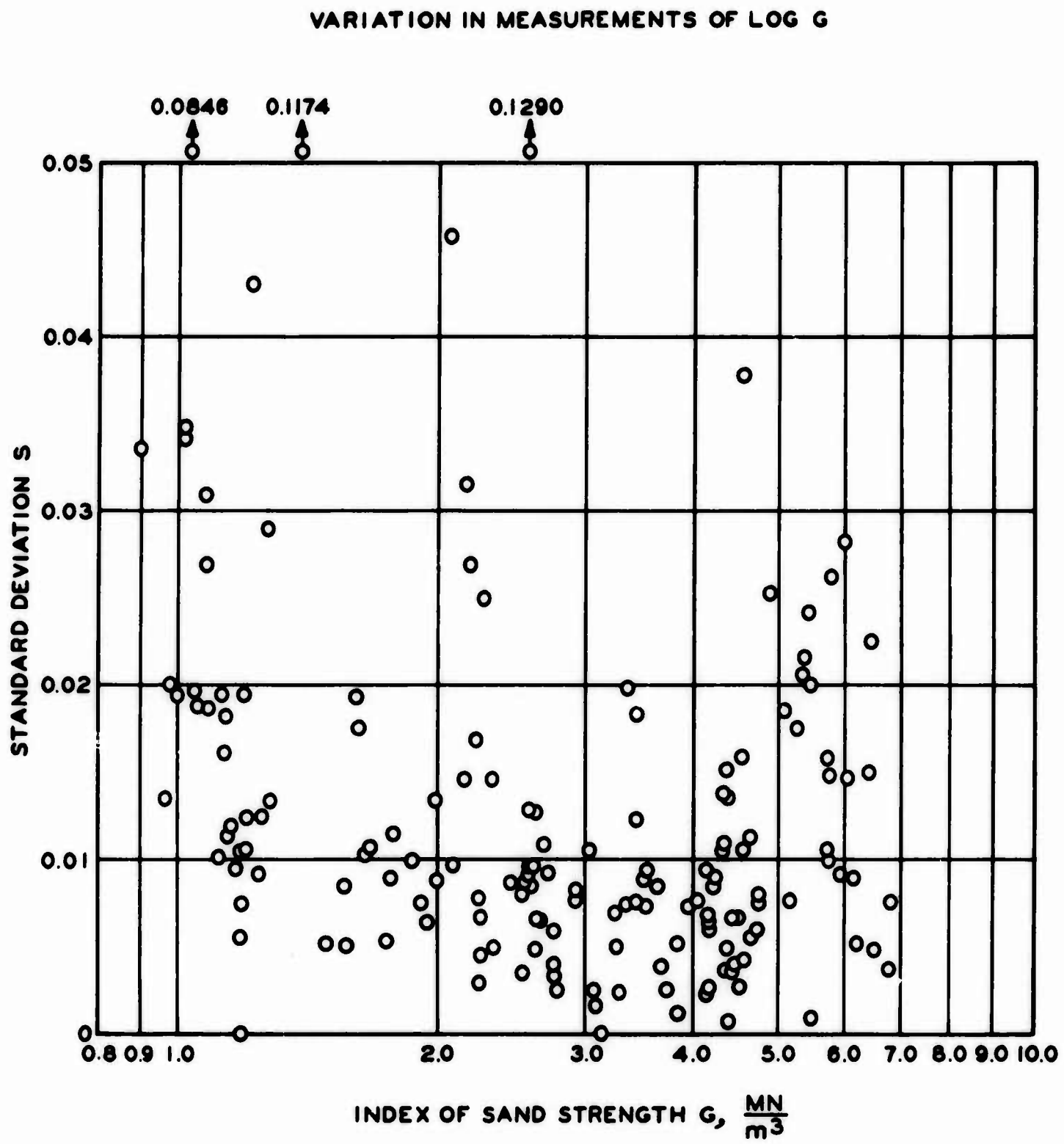
FIGURE 4.12

test beds produced the value $s^2 = 0.000242236$ and $s = 0.0156$.

The covariance of $\dfrac{a_0}{a_1}$ and G is quite difficult to determine. For constant 20 percent slip tests of the type reported herein, a single value of P_{20} is developed within a given test bed, so that there is no possibility of a regression of $\dfrac{a_0}{a_1}$ versus $\log G$ for a given test bed. Values of $\dfrac{a_0}{a_1}$ are 1.4, 1.0, 0.7, 0.9, 1.3, 0.9 for G values in subgroups 8-13, respectively, of Table 4.2. Thus, no strong association is indicated between $\dfrac{a_0}{a_1}$ and G . To be very conservative, $V\left(\dfrac{a_0}{a_1} + \log k\right)$ could be estimated as $V\left(\dfrac{a_0}{a_1}\right) + V(\log G)$ $+ 2 \operatorname{cov}\left(\dfrac{a_0}{a_1}, \log G\right)$, where $\operatorname{cov}\left(\dfrac{a_0}{a_1}, \log G\right)$ is bounded above by $\sqrt{V\left(\dfrac{a_0}{a_1}\right)} \times \sqrt{V(\log G)} = (0.1045)(0.0156) = 0.00163$ (with $\sqrt{V\left(\dfrac{a_0}{a_1}\right)}$ $= \dfrac{0.20483}{1.960}$, as indicated by the 95 percent confidence limits on $\dfrac{a_0}{a_1}$). Then, standard deviation of $\left(\dfrac{a_0}{a_1} + \log k\right)$ is estimated as $\sqrt{0.01442} = 0.120$, and 95 percent confidence limits on $E(\log W_I)$ are $\left(\dfrac{a_0}{a_1} + \log k\right) \pm 1.960(0.120)$. For the example of Figure 4.10, $\left(\dfrac{a_0}{a_1} + \log k\right) \pm 1.960(0.120) = (1.26638 + 1.45076) \pm 0.23520$ $= 2.48194$ to 2.95234 . The range covered by the 95 percent confidence limits on $E(\log W_I)$ is relatively small; however, when corresponding values for $E(W_I)$ are considered (303.3 kN and 896.1 kN, respectively), it is obvious that only very broad estimates can be made of W_I in terms of $10^{a_0/a_1}$ and k . This should not be too surprising when one considers the wide spread in X-axis values as opposed to Y-axis values in the relation of Figure 4.2, on which equation [4.C.7] is based. Load W is included in both the x- and y-axis terms of Figure 4.2; it is not until W is expressed as a function of parameters having estimated

variation (as is done for $\log W_I$ in equation [4.C.7]), that estimates of the value of load that correspond to a particular level of P_{20} can be determined.

While determination of the load that will produce zero pull is an important aspect in the overall description of the soil-track system, immobilization due to overload in a flat, sandy soil appears not to be a practical problem for a track that has its RCG location at its longitudinal center line, as was the case for all tests reported herein. Neither the immobilization nor the maximum pull condition $\left(\text{developed at } \dfrac{P_{20}}{W} = 0.0713 \pm 0.00476\right)$ were achieved in any test reported herein; this, in spite of the fact that the model tracks were tested to values of ground pressure $\dfrac{W}{b\ell}$ up to 132 kN/m^2, well in excess of the approximate upper limit 100 kN/m^2 of existing tracked vehicles.

The P_{20} versus W relation illustrated by example in Figure 4.11 is considered quite reasonable, since a very similar P_{20} versus W relation substantiated by test data at both the maximum pull and zero pull conditions is reported by Turnage [1971] for pneumatic tires in sand. In a very broad general sense, then, pull at 20 percent slip for a track with RCG at its center line can be considered to behave in fashion similar to that of a very large pneumatic tire, as might be expected from a consideration of the sand-tire and sand-track contact patches.

CHAPTER V

SUMMARY AND CONCLUSIONS

In Chapter I it was pointed out that designers of tracked vehicles
need a quantitative description of the track-soil system. The complexity
of this system requires that the initial stage of its study be accom-
plished in a laboratory environment, where close control and systematic
programming of values of the soil and track variables are possible. It
was proposed that, in this thesis, data obtained from testing model
tracks in air-dry sand in the laboratory facilities of the U. S. Army
Engineer Waterways Experiment Station be analyzed to develop a means of
predicting the pull of prototype tracks operating at 20 percent slip
(i.e. P_{20} or near-maximum pull) on level, air-dry sand.

In Chapter II the effects on P_{20} of 16 track and sand vari-
ables selected to provide a reasonably comprehensive description of the
track-sand system were estimated by analysis of three Plackett-Burman
test designs. This type of design requires a very small number of tests
at the expense of a high degree of fractionation, maintains independence
of the estimated effects, and estimates the error variance by a pooled
estimate of variance of dummy parameters. The influence of each of the
16 system variables of the three Plackett-Burman designs was judged on
the basis of (a) their $\underline{t}$-statistic values, (b) their values of
$\dfrac{\left|\text{effect on } P_{20}\right|}{\text{average value of } P_{20}}$, and (c) the deviation of their main effects from

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a straight line on a half-normal plot. Based on the overall analysis
from the Plackett-Burman designs, eight variables were judged to affect
P_{20} significantly: index of sand strength G , load W , track width
b , track length ℓ , pressure in road bogies, road-wheel spacing,
track-frame trim angle θ , and location of drive sprocket. Existing
information with respect to the manner in which the last four variables
influence track pull performance allows recommendations that (a) road
bogies be designed as flexible as practicable, (b) road-wheel spacing
be minimized, (c) trim angle be kept small by locating the at-rest cen-
ter of gravity (RCG) at or forward of the track's geometric center line,
and (d) the track be rear-sprocket driven. Four variables--G , b ,
ℓ , and W--were selected as the basic sand-track variables to be con-
sidered in this study.

In Chapter III principles of similitude were used to develop
three dimensionless, independent Pi terms--$\dfrac{P_{20}}{W}$, $\dfrac{b}{\ell}$, and $\dfrac{G\ell^3}{W}$--to ex-
press the functional relation among P_{20} , G , b , ℓ , and W .
Laboratory test data were analyzed to demonstrate that the second and
third Pi terms can be combined to $\dfrac{G(b\ell)^{3/2}}{W}$.

In Section A of Chapter IV, a simple linear regression determined
the least-squares fit of $\dfrac{P_{20}}{W}$ to $\log \dfrac{G(b\ell)^{3/2}}{W}$ as $\dfrac{\widehat{P_{20}}}{W} = 0.204482$

$+ \; 0.161470 \log \dfrac{G(b\ell)^{3/2}}{W}$ for 158 laboratory test data points. Ninety-
five percent confidence limits for $E\left(\dfrac{P_{20}}{W}\right)$ covered a near-constant
y-axis range of $\left(\dfrac{P_{20}}{W}\right)_{predicted} \pm 0.065$. A plot of residuals on normal
probability paper indicated a normal distribution centered on zero.

To test whether $\dfrac{G(b\ell)^{3/2}}{W}$ adequately reflects the influence of

G , b , ℓ , and W on $\dfrac{P_{20}}{W}$, the 158 data points were separated into

subgroups according to (a) six combinations of b and ℓ , (b) six

levels of G , and (c) nine levels of W . For each data subgroup, a

regression of the form $\dfrac{\hat{P}_{20}}{W} = a_0 + a_1 \log \dfrac{G(b\ell)^{3/2}}{W}$ was developed. The

coefficients of these equations were then compared to A_0 and A_1

from corresponding equation $\dfrac{\hat{P}_{20}}{W} = A_0 + A_1 \log \dfrac{G(b\ell)^{3/2}}{W}$, based on all

test data minus that from the subgroup of interest. The hypothesis

$H_0 : E(a_0) = E(A_0)$ was rejected in five cases, and $H_0 : E(a_1) = E(A_1)$

in one case. The narrowest track and the lowest strength sand test sec-

tions appeared to cause values of a_0 different from those obtained for

the preponderance of the test data.

A visual study was made of changes in values of a_0 , a_1 , and

s (standard error of estimate) for regressions based on the 21 data sub-

groups. For data separated by levels of either G or W , no system-

atic patterns were noted. Values of a_0 and a_1 for data separated by

combinations of b and ℓ were each analyzed as responses in a two-

factor factorial design. In these analyses, MS residuals were de-

scribed by pooled estimates of variances of a_0 and a_1 , respectively.

Hypotheses that b , ℓ , and $b\ell$ have no effect on a_0 were not re-

jected at the 5 percent significance level. Corresponding hypotheses

for a_1 were not rejected for b and $b\ell$, but was rejected for ℓ .

Further examination of values of a_0 and a_1 in the two-factor fac-

torial format showed that differences among values of a_0 and a_1 were

considerably reduced by omitting data for the narrowest track, width

b = 15.2 cm. This raised some suspicion that b = 15.2 cm may be near

the limit below which relations for a model track cannot be extrapolated

to a prototype.

The above analyses developed no clear indication of how exponents

of b and ℓ might be changed from 1.50 to improve the ability of

$\dfrac{Gb^{x_1}\ell^{x_2}}{W}$ to predict $\dfrac{P_{20}}{W}$. Exponents x_1 and x_2 in the dimensionless

prediction term $\dfrac{Gb^{x_1}\ell^{x_2}}{W}$ were varied from 0 to 3, and linear regres-

sions of form $\dfrac{\hat{P}_{20}}{W} = a_0 + a_1 \log \dfrac{Gb^{x_1}\ell^{x_2}}{W}$ were obtained. Values of

standard error s indicated the optimum form of the prediction term

to be $\dfrac{G(b\ell)^{3/2}}{W}$.

In Section B of Chapter IV, no appreciable difference was noted

between two equations in regard to their ability to relate $\dfrac{P_{20}}{W}$ to

G , W , b , and ℓ . These equations were: $\dfrac{\hat{P}_{20}}{W} = 0.204482$

$+ 0.161470 \log \dfrac{G(b\ell)^{3/2}}{W}$ $\left(\text{based on a simple linear regression of}\right.$

$\dfrac{P_{20}}{W}$ on $\log \dfrac{G(b\ell)^{3/2}}{W}\Big)$ and $\dfrac{\hat{P}_{20}}{W} = 0.177491 + 0.141601 \log G$

$- 0.169701 \log W + 0.248757 \log b + 0.271217 \log \ell$ (obtained by a mul-

tiple linear regression of $\dfrac{P_{20}}{W}$ on $\log G,, \quad \log W , \quad \log b ,$

$\log \ell$). This conclusion was based primarily on an F-test on the re-

duction in SS_{res} caused by estimating five coefficients in the mul-

tiple linear regression versus two in the simple linear regression.

These two equations were also transformed to the same general format,

$$\left(\frac{\hat{P}_{20}}{W} - \frac{\overline{P_{20}}}{W}\right) = b_1(\log G - \overline{\log G}) + b_2(\log W - \overline{\log W}) + b_3(\log b - \overline{\log b})$$

$+ b_4(\log \ell - \overline{\log \ell})$, and values of each of b_1 , b_2 , b_3 , and b_4

from the simple and multiple linear regressions were found to be very

nearly equal. Thus, the comparisons of Section B, Chapter IV, gave no

indication that the exponents of G , W , b , and ℓ in $\dfrac{G(b\ell)^{3/2}}{W}$

should be changed, or that significant improvements in the ability to

predict $\dfrac{P_{20}}{W}$ would result from altering the form of the equation

$$\frac{\hat{P}_{20}}{W} = 0.204482 + 0.161470 \log \frac{G(b\ell)^{3/2}}{W} \ .$$

Of particular interest in the description of track pull in sand

are the relations among P_{20} , G , b , ℓ , and W for the maximum

pull and zero pull conditions. In Section C of Chapter IV, an equation

of form $\dfrac{\hat{P}_{20}}{W} = a_0 + a_1 \log \dfrac{G(b\ell)^{3/2}}{W}$ was examined to obtain expressions

or P_{20} , W , and $\dfrac{P_{20}}{W}$ for the maximum pull and zero pull conditions

in terms of a_0 , a_1 , k , and e (where $k = G(b\ell)^{3/2}$). Ninety-five

percent confidence limits were obtained for (a) the value of $\dfrac{P_{20}}{W}$ that

produces maximum P_{20} , and (b) the values of load W_I that produces

$P_{20} = 0$.

Overall, this investigation accomplished its primary goal of

determining a useful quantitative relation between track pull at

20 percent slip and four independent sand and track variables that have

considerable influence on track pull. The adequacy of prediction equa-

tion $\dfrac{P_{20}}{W} = 0.204482 + 0.161470 \log \dfrac{G(b\ell)^{3/2}}{W}$ was examined by several

techniques, each of which indicated that no significant improvement resulted by altering the form of this equation.

Future work should include an expansion of basic-variable diminsionless prediction term $\frac{G(b\ell)^{3/2}}{W}$ to include functions of other independent sand-track variables that influence track performance significantly. Also the number of dimensionless, dependent track performance terms to be predicted should be increased to include measurements of track sinkage and torque input required. Finally, work parallel to that described in this thesis and to that outlined in the above sentences of this paragraph for tracks operating in sand should be accomplished for tracks operating in fine-grained soils (clays and silts).

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